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Interactive optimization of mechanical systems with multiple performance requirements

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Somayajula, Gopichand, Ph.D.

Iowa State University, 1989

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**Interactive optimization of mechanical systems with
multiple performance requirements**

by

Gopichand Somayajula

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
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DOCTOR OF PHILOSOPHY

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In Charge of Major Work

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For the Graduate College

Iowa State University
Ames, Iowa
1989

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DEDICATIONS

To my teachers, Amma and Masterbabu.

1 INTRODUCTION

The goal of structural optimization is to facilitate the design of a product which satisfies a number of performance requirements. The formulation of the problem, which requires engineering judgement, is a cornerstone of the optimization process. The formulation includes the establishment of the conceptual design, prescription of the design requirements, construction of the mathematical model, and description of the loading conditions.

This thesis presents an interactive technique to obtain a solution for multi-objective optimization problems. The method replaces the vector optimization normally associated with multi-objective problems with a scalar optimization under interactive control of the designer. The designer's role is to mitigate conflicts in objectives using engineering judgement.

Chapter 2 presents a review of recent literature in structural optimization and design sensitivity methods. Chapter 3 describes the formulation of a scalar optimization problem which uses the constraint functions to enable the designer to interactively mitigate conflicting design objectives. Thus the repeated minimization of single objective problems yields the solution to the multi-objective problem which is optimal with reference to the designer's preference among the various performance criteria. Chapter 4 presents the step-by-step procedure involved in the overall optimization task. This chapter also describes the OPTIMIZE software which was developed to solve multi-objective optimization problems using the procedures discussed in Chapter 3.

Chapter 5 presents two example problems which illustrate the optimization procedure. The first example concerns an automotive engine block which is redesigned to attain lower mass while shifting the block's natural frequencies away from excitation frequency bands. In the second example, the mounting structure of a heavy-duty truck's exhaust pipe is redesigned to attain lower mass while satisfying displacement, stress, and natural frequency requirements. The final chapter presents conclusions and recommendations for the extension of this work.

2 LITERATURE REVIEW

Structural design problems include the determination of the dimensions of components of a system such that the product fulfills a number of requirements at reasonable cost. The requirements typically involve limits on the local deflections, stresses at various locations in the structure, and natural frequencies of the system.

In a traditional design procedure, the designer formulates a mathematical model, often a finite element model, and then makes a series of repeated changes in the parameters characterizing the model. These parameters may include dimensions, material properties, or support conditions. Each change is followed by analyses carried out to determine whether the structure fulfills the requirements and is reasonable in cost. Usually the proposed design changes are decided based on intuition, experience, and the results from previous analyses.

Structural optimization methods offer an alternative to the traditional approach. Optimization methods attempt to produce the acceptable designs through systematic modifications determined using a mathematical algorithm, speeding up the design process by eliminating inefficiencies in the traditional trial-and-error approach.

Optimization methods have been investigated for optimal design of structures since 1960. Schmit [1]¹ presented the concept of combining structural analysis with numerical optimization to provide a fully automated design tool. Since that time, the developments in modelling, sensitivity analysis, and computer technology have made numerical

¹Numbers presented in brackets designate references in the bibliography.

optimization a powerful and efficient design tool in industrial use. The following review discusses the recent developments in optimization methods and sensitivity analysis techniques relevant to the present problem.

2.1 Design Optimization

The finite element method has proven to be a practical tool for analyzing complex structures. The governing equilibrium equation for a finite element model of a structure subjected to quasistatic load typically has the form:

$$\mathbf{K}(\mathbf{v})\mathbf{x} = \mathbf{f}(\mathbf{v}) \quad (2.1)$$

where \mathbf{K} is the symmetric nonsingular structural stiffness matrix of order n ,

\mathbf{f} is equivalent nodal load vector of order n ,

\mathbf{x} is nodal displacement vector of order n , and

\mathbf{v} is the design parameter vector of order s .

The design parameter vector is also known as design variable vector. The typical design variables are cross-sectional dimensions, panel thicknesses, etc. The order n corresponds to the degrees of freedom of the model and order s refers to the number of design variables.

The free vibration response of the structure is typically governed by an eigenvalue problem:

$$\mathbf{K}(\mathbf{v})\phi_i = \lambda_i \mathbf{M}(\mathbf{v})\phi_i \quad (2.2)$$

where \mathbf{M} is the symmetric nonsingular structural mass matrix of order n ,

λ_i is the eigenvalue related to the i th natural frequency,

ϕ_i is the associated eigenvector.

Equation (2.1) and (2.2) yield the displacements, stresses, and natural frequencies of the structural system. If equation (2.1) indicates that the design does not satisfy either the strength requirement or stiffness requirements, or if equation (2.2) indicates that it has an undesirable frequency spectrum, it is appropriate to make design changes in \mathbf{v} and then re-solve the equations to re-evaluate the design. Numerical optimization schemes help determine the design changes in a systematic fashion.

Numerical optimization refers to a computer optimization program which determines the values for the design variables to maximize or minimize a specific function known as the objective (or criterion, or merit, or cost) function, while satisfying a set of design requirements called constraints. For linear elastic structural systems, the design optimization problem can be formulated as follows:

$$\begin{aligned}
 &\text{Minimize:} && \mathbf{C}(\mathbf{v}) \\
 &\text{Subject to:} && d_i(\mathbf{v}) \leq \bar{d}_i \quad i = 1, \dots, p \\
 & && s_j(\mathbf{v}) \leq \bar{s}_j \quad j = 1, \dots, q \\
 & && f_k \notin (f_k^l, f_k^u) \quad k = 1, \dots, r \\
 & && v_m^l \leq v_m \leq v_m^u \quad m = 1, \dots, s
 \end{aligned} \tag{2.3}$$

where \mathbf{C} is the objective function,
 \mathbf{v} is the design variable vector of order s ,
 d_i is the displacement at node i ,
 \bar{d}_i is the limit on the displacement at node i ,
 s_j is the stress in element j ,
 \bar{s}_j is the limit on the stress in element j ,

f_i are the natural frequencies of the system,
 (f_k^l, f_k^u) are the undesirable frequency bands, and
 (v_m^l, v_m^u) are the limits on the design variables.

The bounds (v_m^l, v_m^u) on the design variables are imposed to restrict the sizes of the design variables due to considerations such as manufacturing limitations, physical practicability, aesthetics, etc.

Traditional optimization techniques consider either weight or cost of the structure as the objective and the other performance requirements such as displacements, stresses, and system natural frequencies, as constraints in the problem formulation. In this approach, system performance becomes a sub-goal of the overall design, and hard and fast bounds must be specified for the constraints before hand. Numerous solution methods are available for this type of problems.

Schmit's 1981 review[2] covers the 20 years of the various developments in structural optimization stressing mathematical programming techniques and optimality criteria. Mathematical programming techniques and the optimality criteria techniques begin with the same design problem. However, in the optimality criteria approach, rather than directly minimizing the objective function, one specifies a criterion such that, if it is satisfied, subject to the constraints, then the design is defined as optimum. A common criterion is that the strain energy density will be same everywhere in the structure. On the other hand, mathematical programming offers a general tool to solve any nonlinear-constraint-minimization problem. Lev[3], Vanderplaats[4], and Levy[5] provide a comprehensive review of structural optimization and the contributions of numerical techniques to the developments of structural optimization.

The view taken in this thesis is this: Although the minimization of weight is a common objective which is reasonable and well accepted, it would be more desirable in some cases if, in addition to reducing the weight, the objective were to hold other responses, such as displacements and stresses close to a small value, and natural frequencies away from undesirable bands. In this context the optimization problem would consider the minimization of the performance indices and weight as multiple objectives. Then each performance requirement of Problem (2.3) would be expressed as some function of the corresponding response variable to formulate performance indices. The optimal structure should then be determined based on the trade-offs among the objectives. We classify this kind of problem representation under multiobjective optimization. The labels vector optimization or multicriteria optimization are appropriate. The multiobjective optimization problem, in formal terms, is as follows:

$$\begin{aligned} \text{Minimize: } & [W(\mathbf{v}), D_i(\mathbf{d}), S_j(\mathbf{s}), F_k(\mathbf{f})] \\ \text{Subject to: } & v_m^l \leq v_m \leq v_m^u \quad m = 1, \dots, s \end{aligned} \quad (2.4)$$

where W is the weight of the structure,
 D_i is the displacement objectives,
 S_j is the stress objectives,
 F_k is the frequency objectives,
 \mathbf{d} is the displacement vector,
 \mathbf{s} is the stress vector,
 \mathbf{f} is the natural frequency vector, and
 (v_m^l, v_m^u) are the limits on the design variables.

As mentioned earlier, in multiobjective optimization there exists a possibility for trade-off among criteria, i.e., a change in design may result in improvement according to one or more criteria but only at the expense of a degradation as measured by others. One approach to derive the solution for the multiobjective problem is by using Pareto-optimization[6]. A design among admissible designs is called Pareto optimal if there exists no other admissible design that will yield an improvement in one objective without the worsening of at least one criterion. A given multiobjective optimization problem may have any number of Pareto solutions, and the solution to the multiobjective problem is included among them. In a strictly Pareto approach, the full set of optimal solutions must be determined first. Then the optimal design is identified based on the designer's judgement or some other basis of choice. Several structural applications of Pareto optimization are reported in [7,8].

A drawback of developing the Pareto optimal is the extensive calculation required to obtain all possible solutions. Starkey and Bernard[9] presented a related approach known as the constraint functions method to obtain the scalar form and determine the "best" design modifications for shifting the natural frequencies of the system away from the undesirable frequency bands. They represented a constraint function $C(\mathbf{f}, \mathbf{v})$ of the form:

$$C(\mathbf{f}, \mathbf{v}) = AS(\mathbf{v}) + \sum_{i=1}^n B_i F_i(\mathbf{f}) \quad (2.5)$$

where \mathbf{f} is the vector of system natural frequencies,

\mathbf{v} is the vector of design variables,

$S(\mathbf{v})$ is the size constraint function,

$F_i(\mathbf{f})$ is the frequency constraint function, and

A and B_i are weighting factors.

The frequency constraint functions, $F_i(\mathbf{f})$, are scalar functions which are large when the natural frequencies lie in undesirable locations. The size constraint function, $S(\mathbf{v})$, is another scalar function which becomes large when structural changes are large. Figures 2.1 and 2.2 show the characteristics of the frequency and size constraint functions respectively. The coefficients A and B_i are positive scalars which weight the relative importance of natural frequencies and structural changes. This kind of formulation for the objective function assigns higher costs to those designs which produce the less desirable vibration behavior, and require more costly design changes. The minimization of the constraint function yields the so called "best" design. Rizai[10] used the similar approach to improve the dynamic performance of large systems with large design changes.

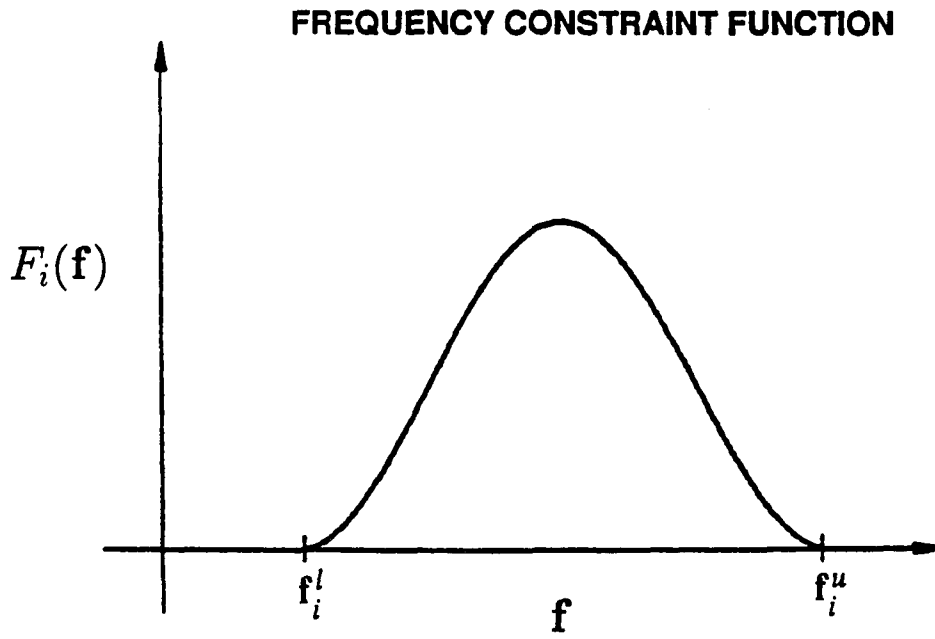


Figure 2.1: Frequency constraint function

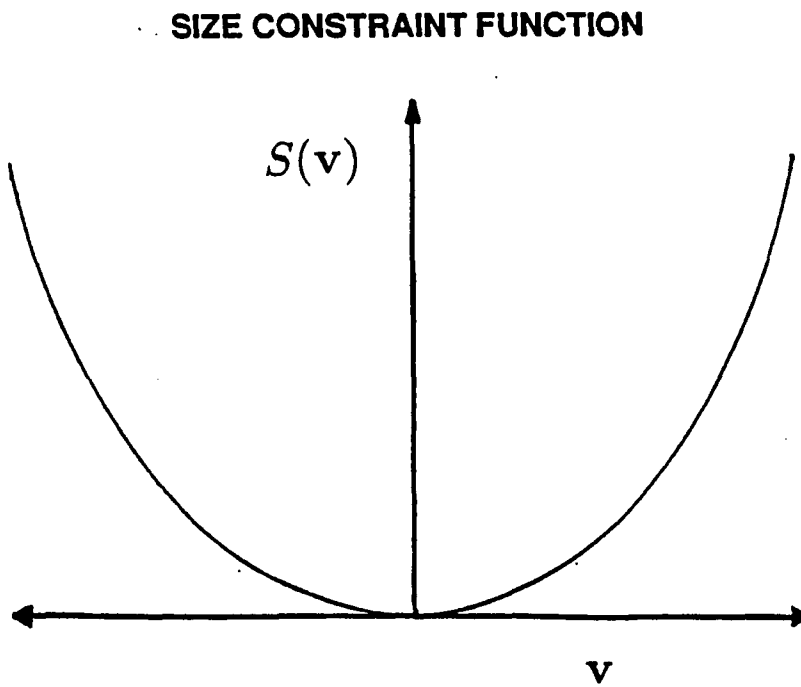


Figure 2.2: Size constraint function

The constraint function approach is mathematically quite similar to sequential unconstrained minimization technique (SUMT)[11]. However, there is a conceptual distinction between these two methods[12]. In SUMT, the inequality and equality constraints are replaced with smooth mathematical functions which become unbounded at (or near) the constraint. The weighted sum of these functions is added to the objective function to form an unconstrained minimization problem. By solving a sequence of unconstrained minimization problems with different weights, the solution to the original constrained optimization problem is obtained. In the constraint function approach, on the other hand, one varies the set of performance constraints to explore the required costs, in terms of design, to achieve the goal. This feature is quite useful for investigating the trade-offs that typically exist between the selection of the constraints and the cost of the design.

The direct solution to the minimization problem of (2.5) is computationally impractical because a large number of eigenvalue solutions are required in the optimization process. Starkey used linear approximation to the eigenvalues to relate the natural frequencies to the design changes and thus converted the problem into a more tractable sequence of explicit approximation problems. However, if the linear approximation is good only within a small region of the design space, a large number of approximate problems may have to be solved to obtain a near-optimum solution. Therefore, the range of validity of the approximation plays a key role in the successful development of an efficient structural optimization methodology. Rizai used higher order Taylor series expansion to expand the useful range of the approximations. His work showed validity for large changes but was limited to one design variable. Woo [13] presented a powerful class of generalized hybrid constraint approximations that require only the first-order constraint function derivatives to overcome the inherent nonlinearity of the frequency constraints for frame-type structures.

The goal of this thesis is to extend Starkey's methodology to develop an interactive design optimization process for large problems having several displacement, stress, and frequency constraints. It recasts the traditional optimization problem as a multiobjective optimization problem using constraint functions. This will allow the designer to explore the cost/benefit trade-offs associated with different designs and to tighten or loosen the design requirements to obtain cost effective designs in an efficient manner.

A major step in this optimization procedure is the calculation of the derivatives of the response quantities - displacements, stresses, and frequencies, with respect to the design variables. Calculation of these derivatives for structural design problems is called design sensitivity analysis. The next section discusses the literature in this area.

2.2 Design Sensitivity Analysis

Design optimization is intimately connected with sensitivity analysis and the cost of calculating sensitivities is an important contributor to the total cost in many optimization procedures. In addition, sensitivity derivatives have several other applications in structural mechanics including approximate analysis (and reanalysis) techniques, analytical model improvement, and assessment of design trends. The literature on this subject is so voluminous that it is difficult to cite all available references. A review of the state of the art in sensitivity analysis is contained in survey papers[14,15], and a monograph[16].

There are various approaches to obtain the desired sensitivity information for structural design problems. The variational approach[17,18], which is particularly well-suited to continuum structures, borrows the concept of material derivative from continuum mechanics. The finite difference method[19,20], direct or design space method[21], and adjoint variable or state space method[22] are quite popular for structural models

derived through finite elements. In finite difference method, the original response and the modified response are computed by repeating the analysis for a perturbation in design variable. The approximate derivative can be calculated using an appropriate difference formula. Analysts tend to avoid finite difference methods due to potential accuracy problems and high computational cost.

Both direct and adjoint methods produce the desired sensitivity information. The essential difference is in the computational effort needed as a result of the order of the matrix operations. If the number of response quantities (constraints) is small compared to the number of design variables, the adjoint method is usually best. If the converse is true, the direct method is preferred[23]. In this section, the basic theory underlying the displacement, stress, and natural frequency sensitivity analyses using the direct method is briefly explained.

2.2.1 Displacement sensitivities

The governing equation for displacements due to a static load is given by equation(2.1). Differentiation of equation (2.1) with respect to the design variables, v_i , yields:

$$\mathbf{K} \frac{d\mathbf{x}}{dv_i} = \frac{\partial \mathbf{f}}{\partial v_i} - \frac{\partial \mathbf{K}}{\partial v_i} \mathbf{x} \quad (2.6)$$

The applied load, \mathbf{f} , is usually independent of the design variables, thus we assume $\frac{\partial \mathbf{f}}{\partial v_i}$ to be zero. The remaining quantity on the right hand side of Equation (2.6) is obtained by multiplying $\frac{\partial \mathbf{K}}{\partial v_i}$ by the displacement vector, \mathbf{x} . Equation (2.6) can be solved by the same solution algorithm used for equation (2.1), taking advantage of the fact that \mathbf{K} is available in factored form from the solution of equation (2.1). The desired sensitivity can therefore be determined in $\mathbf{O}(n^2)$ calculations.

2.2.2 Stress sensitivities

The computation of stress sensitivities is straightforward once the displacement sensitivities are computed. In finite element analysis, the element stresses, σ , are related to the displacements by the equation:

$$\sigma = \mathbf{S}\mathbf{x} \quad (2.7)$$

where σ is the element stress vector,

\mathbf{x} is the nodal displacement vector, and

\mathbf{S} is the stress-displacement matrix.

Sensitivity of stresses to design changes can be obtained by differentiating equation (2.7) with respect to the design variable v_i :

$$\frac{d\sigma}{dv_i} = \mathbf{S} \frac{d\mathbf{x}}{dv_i} + \frac{\partial \mathbf{S}}{\partial v_i} \mathbf{x} - \frac{\partial \mathbf{G}}{\partial v_i} \mathbf{T} \quad (2.8)$$

For finite elements such as rods, memberanes, and shear panels, \mathbf{S} is independent of \mathbf{v} and stress sensitivities are obtained by substituting $\frac{d\mathbf{x}}{dv_i}$ into the Equation (2.8). For bending type elements, \mathbf{S} may be a function of the design variable \mathbf{v} , and $\frac{\partial \mathbf{S}}{\partial v_i}$ has to be evaluated before using equation (2.8).

2.2.3 Natural frequency sensitivities

Equation (2.2) is typically used for analysis of structural vibration. Assuming that the mode shapes are normalized with respect to the mass matrix \mathbf{M} ,

$$\phi_i^T \mathbf{M} \phi_i = 1 \quad (2.9)$$

Following reference 24, we diffrentiate equation (2.2) with respect to design variable v_i

$$\frac{\partial \mathbf{K}}{\partial v_i} \phi_i + \mathbf{K} \frac{\partial \phi_i}{\partial v_i} - \frac{d\lambda}{dv_i} \mathbf{M} \phi_i - \lambda_i \frac{\partial \mathbf{M}}{\partial v_i} \phi_i - \lambda_i \mathbf{M} \frac{\partial \phi_i}{\partial v_i} = 0 \quad (2.10)$$

After premultiplying by ϕ_j^T and substituting Equation (2.9), Equation (2.10) can be simplified to:

$$\frac{d\lambda}{dv_i} = \phi_i^T \left[\frac{\partial \mathbf{K}}{\partial v_i} - \lambda_i \frac{\partial \mathbf{M}}{\partial v_i} \right] \phi_i \quad (2.11)$$

$\frac{\partial \mathbf{K}}{\partial v_i}$ and $\frac{\partial \mathbf{M}}{\partial v_i}$ have to be evaluated analytically or using finite difference methods. The next chapter describes the formulation of the optimization problem using this sensitivity information.

3 OPTIMIZATION METHODOLOGY

Once the characteristics of the proposed design are estimated using a mathematical model, the next step is usually the determination of the best dimensions of the product to satisfy a specified set of performance criteria. In a finite element context, these performance criteria can include displacements of the model at various nodes and the stresses of the model at various elements due to different loading conditions, and the natural frequencies and mode shapes of the model. The desirability of certain performance combinations will dictate which design changes are best for the designer's situation. Often it is straightforward to find a design which satisfies one performance specification, but it may be difficult to meet multiple performance requirements. The "best" design will then be a compromise which attempts to satisfy all the performance requirements.

Traditional optimization methods set their goal on a single objective, thus forcing system performance to be a sub-goal of the overall design procedure. By expressing multiple performance requirements as part of the objective function whose minimization yields the desired performance, the goal of the optimization can focus on the system performance. The performance requirements may include several criteria such as weight, cost, stiffness, and deflection, the problem is then multi-criteria optimization problem. The criteria may be commensurable (measured in the same units) or noncommensurable (measured in different units).

One approach to determine the solution for the multi-objective problem is by using

Pareto-optimization technique[25]. In this thesis, on the other hand, the multicriteria optimization problem is reduced to a scalar optimization problem using the constraint function technique[9]. This yields a feasible way to find a compromise solution through an interactive optimization procedure. During such an interactive session the designer is asked to provide his preferences concerning the importance of the various criteria. These preferences are expressed by means of weighting coefficients assigned to those criteria. Higher value for a weighting coefficient increases the importance of the corresponding performance criterion.

In the interactive use of this method, two phases are distinguished, the computation and decision phases. In the computation phase, an optimal solution is found for the given values of weighting coefficients. In the decision phase, the designer decides whether or not the present solution is optimal with respect to his preferences. If not, he must give new values of the weighting coefficients and a new computational phase is initiated. The procedure is stopped when a subjectively optimal solution is found.

3.1 Problem Formulation

The constraint function technique presented in [9] extended here to incorporate trade-offs between structural design variables and multiple structural performance requirements. The constraint functions are smooth curves or surfaces which assign a cost penalty to a design. The penalties become larger as the design produces less desirable structural behavior or when it becomes heavier. Using these constraint functions, the individual performance requirements are transformed into a number of objective functions. The objective function for the scalar optimization problem, which is called the "global" objective or cost function, consists of the sum of the weighted single objective functions. The preferences of the design engineer are expressed by means of

the weighting factors. In our design optimization problem, the global cost function $C(W, \mathbf{d}, \mathbf{s}, \mathbf{f}, \mathbf{v})$ is expressed as:

$$C(W, \mathbf{d}, \mathbf{s}, \mathbf{f}, \mathbf{v}) = P \sum_{l=1}^t Z_l(\mathbf{d}, \mathbf{v}) + Q \sum_{i=1}^p D_i(d_i) + R \sum_{j=1}^q S_j(s_j) + U \sum_{k=1}^r F_k(\mathbf{f}) + TW(\mathbf{v}) \quad (3.1)$$

where W is the weight of the design,

\mathbf{d} is a p -vector of desired nodal displacements,

\mathbf{s} is a q -vector of desired element stresses,

\mathbf{f} is a r -vector of natural frequencies,

\mathbf{v} is a g -vector of design variables,

Z_l is the combination constraint function,

D_i is the displacement constraint function,

S_j is the stress constraint function,

F_k is the frequency constraint function, and

P, Q, R, U and T are scalar weighting factors.

The combination constraint functions $Z_l(\mathbf{d}, \mathbf{v})$ are scalar functions which become large when the size and response "combination requirements" do not satisfy the specified limits. When a design requirement is expressed as the function of both the size of the design parameter and the displacement response of the structure, the requirement is called "combination constraint." Consider an example wherein the sum of the displacement and the diameter of a rod which is surrounded by a circular tube is limited by the available clearance around the rod. This kind of requirement can be classified as the combination constraint.

The constraint functions $D_i(d_i)$, $S_j(s_j)$, and $F_k(f_i)$ are scalar functions which become large when the displacements, stresses, and frequencies violate prescribed con-

ditions.

The coefficients P, Q, R, U , and T are positive scalars which weight the relative importance of the combination, displacement, stress, frequency, and weight requirements respectively. The choice for these coefficients determines the relative importance of the response and the weight. For example, to relax the stress requirements, R would be assigned a smaller value. By using an interactive optimization scheme, one can explore the various design alternatives for different weighting coefficients. This process allows the design engineer to look at the cost and performance trade-offs associated with different designs and then use his judgement to decide whether to relax or tighten the constraints to obtain the "best" design for the problem at hand.

The minimization problem based on this transformation of vector optimization into scalar optimization can be expressed as:

$$\text{Minimize: } C(W, \mathbf{d}, \mathbf{s}, \mathbf{f}, \mathbf{v})$$

$$\text{Subject to: } v_m^l \leq v_m \leq v_m^u \quad m = 1, \dots, g \quad (3.2)$$

$$z_n^l \leq z_n \leq z_n^u \quad n = 1, \dots, h \quad (3.3)$$

where v_m and z_n represent the design variables and combination variables respectively. The solution to this problem provides an improvement in the design with reference to the designer's choice of the importance of the constraints.

3.2 Constraint Functions

In this thesis, the constraint functions are chosen to be either quadratic or trigonometric functions. The functions are synthesized in such a way that they become smaller as the design improves. Because they are smooth, derivatives of these functions with respect to the design variables are always well behaved. In order to avoid the possible

difficulty in solving a poorly scaled system, the constraint functions for different kinds of responses are normalized to a common range.

3.2.1 Displacement constraint function

The displacement constraint function is formulated based on the prescribed limits on the displacements at finite element nodes for a given loading condition. The constraint function has the form:

$$D_i(d_i) = \begin{cases} 0 & \text{for } d_i \leq \bar{d}_i \\ \left(\frac{d_i - \bar{d}_i}{d_i}\right)^2 & \text{for } d_i > \bar{d}_i \end{cases} \quad (3.4)$$

where D_i is the displacement constraint function for a specific nodal displacement d_i . The \bar{d}_i is the nominal limit on the displacement value. The shape of this function is shown in Figure 3.1. This function becomes large when the nodal displacement begins to exceed the prescribed limit and assigns zero value for displacements less than the limit value.

3.2.2 Stress constraint function

The stress constraint function is based on the prescribed limits on the stresses for a given loading condition. The constraint function has the form:

$$S_j(s_j) = \begin{cases} 0 & \text{for } s_j \leq \bar{s}_j \\ \left(\frac{s_j - \bar{s}_j}{s_j}\right)^2 & \text{for } s_j > \bar{s}_j \end{cases} \quad (3.5)$$

where S_j is the stress constraint function for a specific element stress s_j . The \bar{s}_j is the limit on the stress value. The shape of this function is shown in Figure 3.2. This function becomes large when the element stress begins to exceed the prescribed limit and assigns zero value for the stresses less than the limit value.

3.2.3 Frequency constraint function

The frequency constraint function is formulated to reflect the goals of the design problem. The function becomes large when the natural frequencies lie in an undesirable frequency range and is set to zero when the frequencies are outside the undesirable range. Natural frequencies near the center of the undesirable range are penalized more than those near the ends. An improved design is indicated when the natural frequencies are shifted away from the center of the range. Definition of this function requires as input the lower and upper values of the undesirable frequency range.

The frequency constraint function can be expressed as a cosine which yields the desired penalty.

$$F_k(f_i) = \begin{cases} (1 - \cos[2\pi(f_i - f_k^l)/(f_k^u - f_k^l)]) & \text{for } f_k^l \leq f_i \leq f_k^u \\ 0 & \text{for } f_i \notin (f_k^l, f_k^u) \end{cases} \quad (3.6)$$

where F_k is the frequency constraint function for a specific undesirable frequency range. The range (f_k^l, f_k^u) delineates an undesirable frequency band. The shape of this function is shown in Figure 3.3.

3.2.4 Combination constraint function

The combination constraint function is formulated based on the prescribed limits on the combined displacements and design variables for a given loading condition. Each requirement is identified by a combination variable z_i . The constraint function has the form:

$$Z_i(z_i) = \begin{cases} 0 & \text{for } z_i \in (z_i^l, z_i^u) \\ \left(\frac{z_i - z_i^u}{z_i}\right)^2 & \text{for } z_i > z_i^u \\ \left(\frac{z_i^l - z_i}{z_i}\right)^2 & \text{for } z_i < z_i^l \end{cases} \quad (3.7)$$

where Z_i is the combination constraint function for a specific combination variable z_i . The range (z_i^l, z_i^u) delineates the limits on the combination variable. The shape of this function is shown in Figure 3.4. This function becomes large when the combination variable begins to violate the prescribed limits and assigns a zero value otherwise.

DISPLACEMENT CONSTRAINT FUNCTION D_i

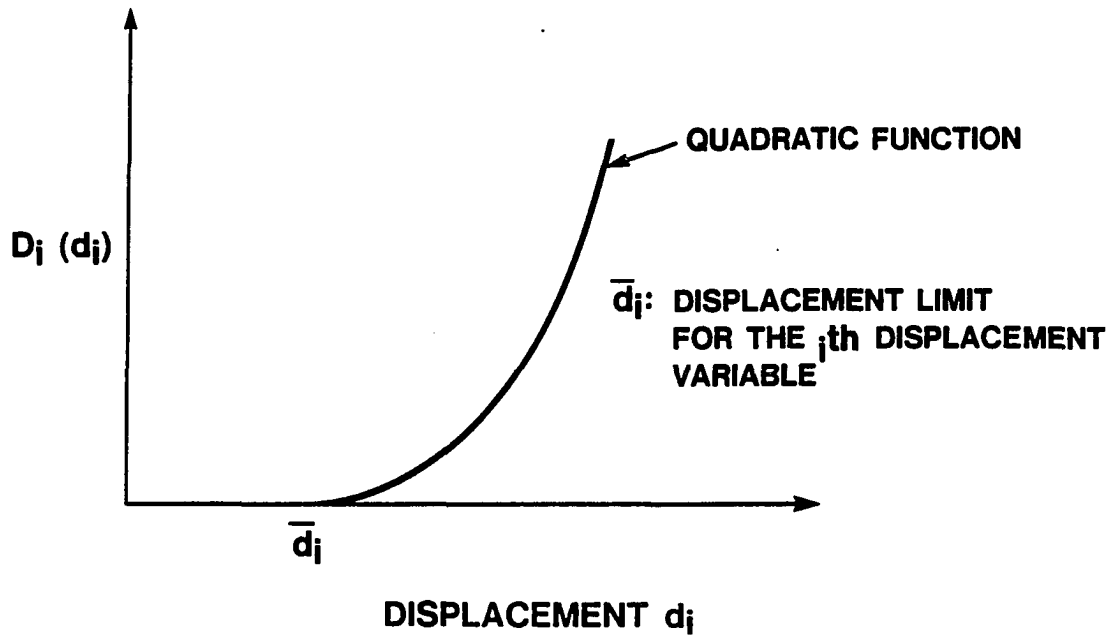


Figure 3.1: Displacement constraint function

STRESS CONSTRAINT FUNCTION S_j

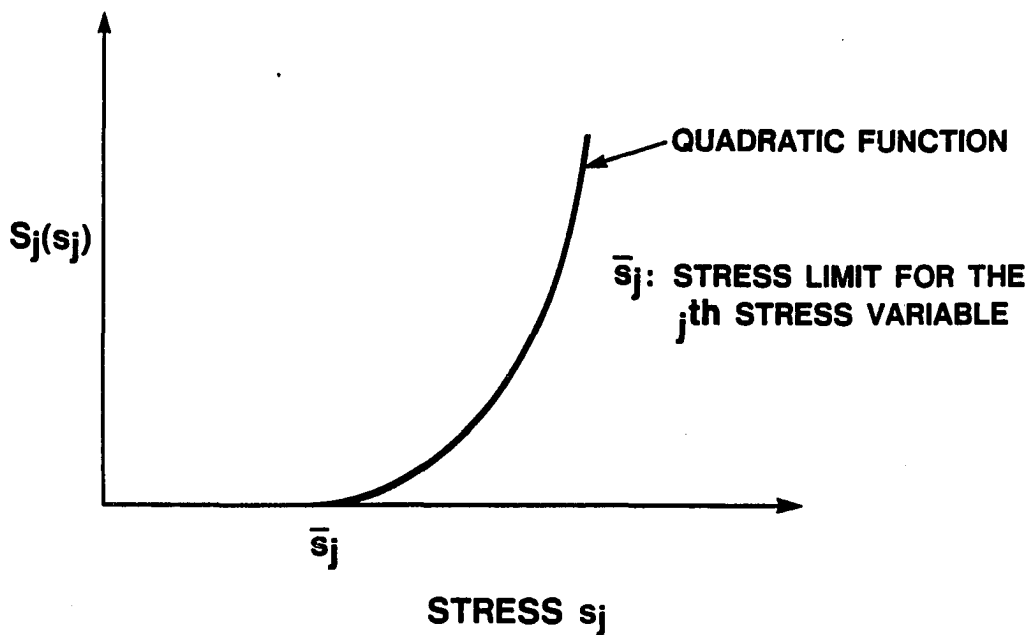


Figure 3.2: Stress constraint function

FREQUENCY CONSTRAINT FUNCTION F_k

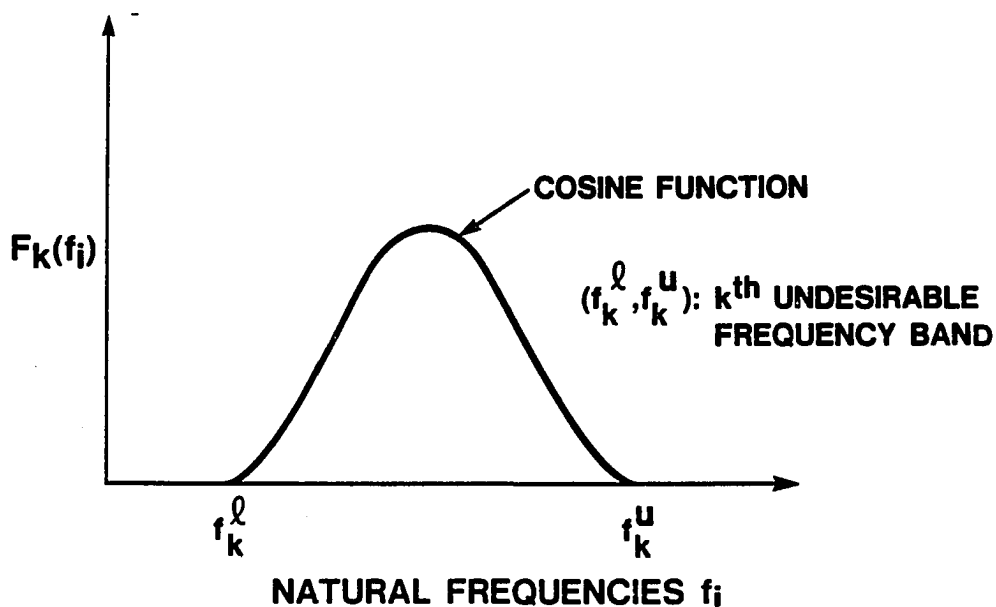


Figure 3.3: Frequency constraint function

COMBINATION CONSTRAINT FUNCTION Z_k

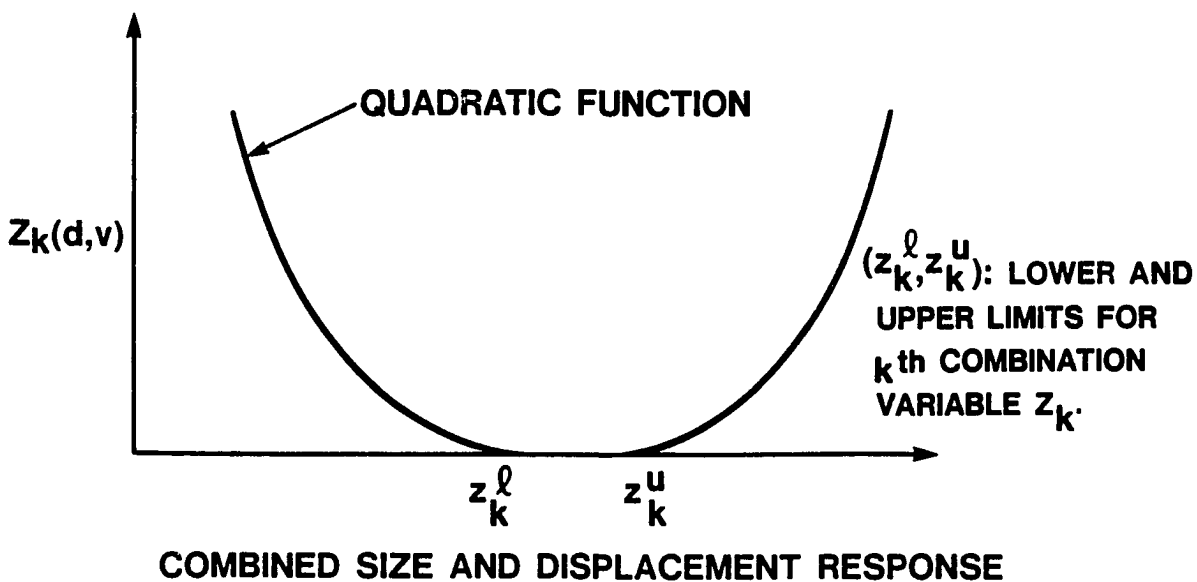


Figure 3.4: Combination constraint function

All these constraint functions are expressed as functions of the response variables d_i, s_j , and f_k rather than design variables v_m . However, in the minimization process, the objective function C has to be evaluated hundreds or perhaps thousands of times for different trial design variable vectors. The exact calculation of the constraint functions, which are nonlinear implicit functions of the design variables, is computationally very burdensome because it invariably involves the solution of equilibrium and eigenvalue problems. One way to alleviate this problem is to use sensitivity methods. This yields an approximate optimization problem which is easily solved. Since the linear approximations inherent in our use of sensitivity has a limited range of validity, the structure is then reanalyzed and a new approximation is created. The process is repeated until it produces an acceptable solution \mathbf{v}^* . In this thesis we refer to this process of setting up the approximate optimization problem and analyzing the resultant design as one iteration. The number of such design iterations required to attain the "best" design depends on the accuracy of the approximations to the solutions of the equilibrium and eigenvalue problems which undergo in the calculation of the objective function.

3.3 Design Variable Linking

The design sensitivities for this formulation are obtained using a direct approach which is implemented in the commercial finite element analysis code NASTRAN[26]. In the direct approach, the sensitivities are computed using implicit differentiation of the equilibrium equations (2.1 and 2.2) with respect to each of the design variables v_m . Thus the computational cost of the method grows with the number of design variables. The number of independent design variables used in the optimization problem can be greatly reduced with an approach we refer to here as design variable linking.

The finite element analysis usually requires a large number of elements to predict

accurately the response quantities. The number of elements is much larger than the number of design variables required for the design problem, so it is not practical to introduce an independent design variable for each finite element.

Design variable linking reduces the number of independent design variables by relating the properties of a set of finite elements to a single design variable through a mapping scheme. Thus one independent variable controls the properties of all the finite elements in that linking group. A typical mapping scheme is of the form:

$$\mathcal{V} = \Gamma \mathbf{v} \quad (3.8)$$

Where \mathcal{V} is the dependent design variable vector that contains the properties of all the finite elements in the analysis model, and \mathbf{v} is the independent design variable vector. Γ is the mapping matrix that relates the independent design variables to the dependent design variables. The designer usually defines this matrix at the beginning of the design phase.

3.4 Move-limit Strategy

Even though we routinely set limits on the design variables based on practical considerations, we need to specify another set of limits for the variables to avoid numerical problems. This is necessary because the sensitivity approximations are valid only for a limited region of the design space. A simple move-limit strategy for the design variable v_m can be specified as:

$$(1 - \alpha)v_m \leq v_m \leq (1 + \alpha)v_m \quad (3.9)$$

where α delineates the permissible change in the design variable for the design iteration.

It seems clear that economy and success of the solution will depend on the choice of move limits. If the move limits are made too tight, i.e., α values are made too small, the convergence will be very slow. If move-limits are too large, oscillations may occur.

Adaptive methods in which the limit to be placed on the next move depends on the results of previous iteration results are quite helpful in improving the efficiency of the move limit specification strategy.

An interactive software package was developed based on the concepts discussed in this chapter to determine the design changes for the optimum performance based on the judgement of the designer. The procedure is explained in the next chapter.

4 OPTIMIZATION PROCEDURE

The design optimization procedure laid out in this thesis requires three components:

- The ability to generate a mathematical model of the structure. For the purpose of illustration and example here we use MSC/NASTRAN.
- The ability to determine sensitivities, for example, the derivative of the stress at some element with respect to a design variable such as plate thickness. Again we use MSC/NASTRAN.
- An optimizer which will minimize an objective function based on the sensitivities. For this purpose, we use CONMIN[27].

Using the concepts of the previous chapter, the overall optimization task may be segmented into the following steps.

1. Preparation of analysis model. (This step generates the finite element model based on the geometry, loading, and boundary conditions and is usually known as pre-processing.)
2. Finite element analysis. (This step computes displacements, stresses, and natural frequencies.)
3. Identification of critical response variables. (This step results from an assessment of the results of step 2.)

4. Preparation of design model that includes an objective function with variable weighting factors and constraints.
5. Evaluation of sensitivities of the critical responses. (This step establishes an approximate relationship between stresses, displacements and natural frequencies and the design variables.)
6. Formulation of the "global" objective function based on designer's choice of weighting factors. (The designer sets the weighting factors P, Q, R, U, and T in Equation 3.1.)
7. Optimization of the approximate problem subject to move limits on the design variables. (This step is interactive because we are using sensitivities rather than recalculating the finite element problem.)
8. If the obtained design does not satisfy the designer's criteria, adjust the weighting factors and go to step(6), and repeat the process from there.
9. Reperform finite element analysis to evaluate the critical responses. (At this step, the designer verifies whether or not the approximate solution based on sensitivities is sufficiently close to the exact solution based on a finite element analysis of the new design.)
10. If the finite element solution does not closely match the sensitivity-based results, the designer may wish to go to step(3) and repeat the process from there.

Figure 4.1 presents the flow chart of the optimization scheme. Some important details of the procedure are presented here.

THE FLOW CHART OF THE O

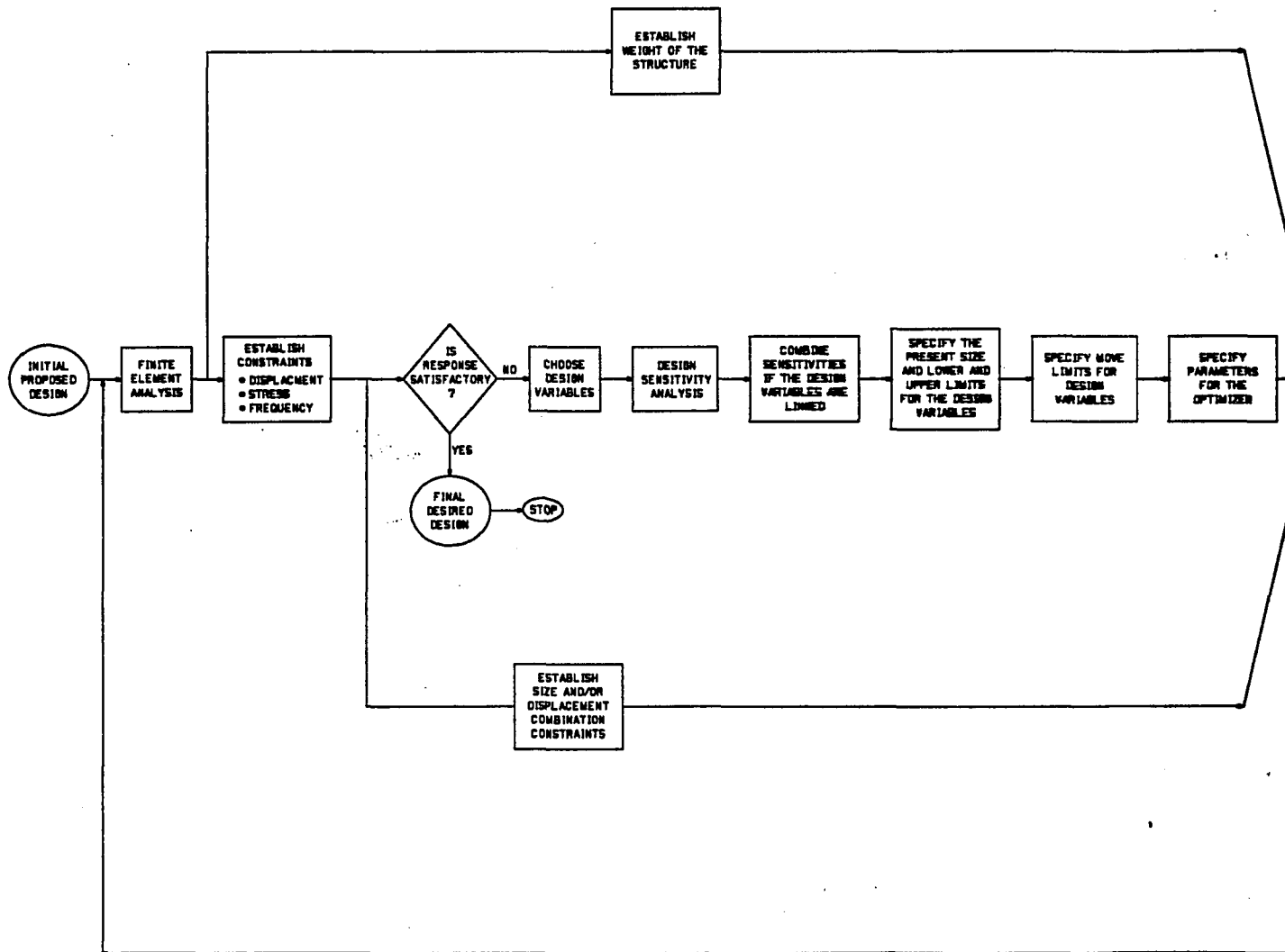
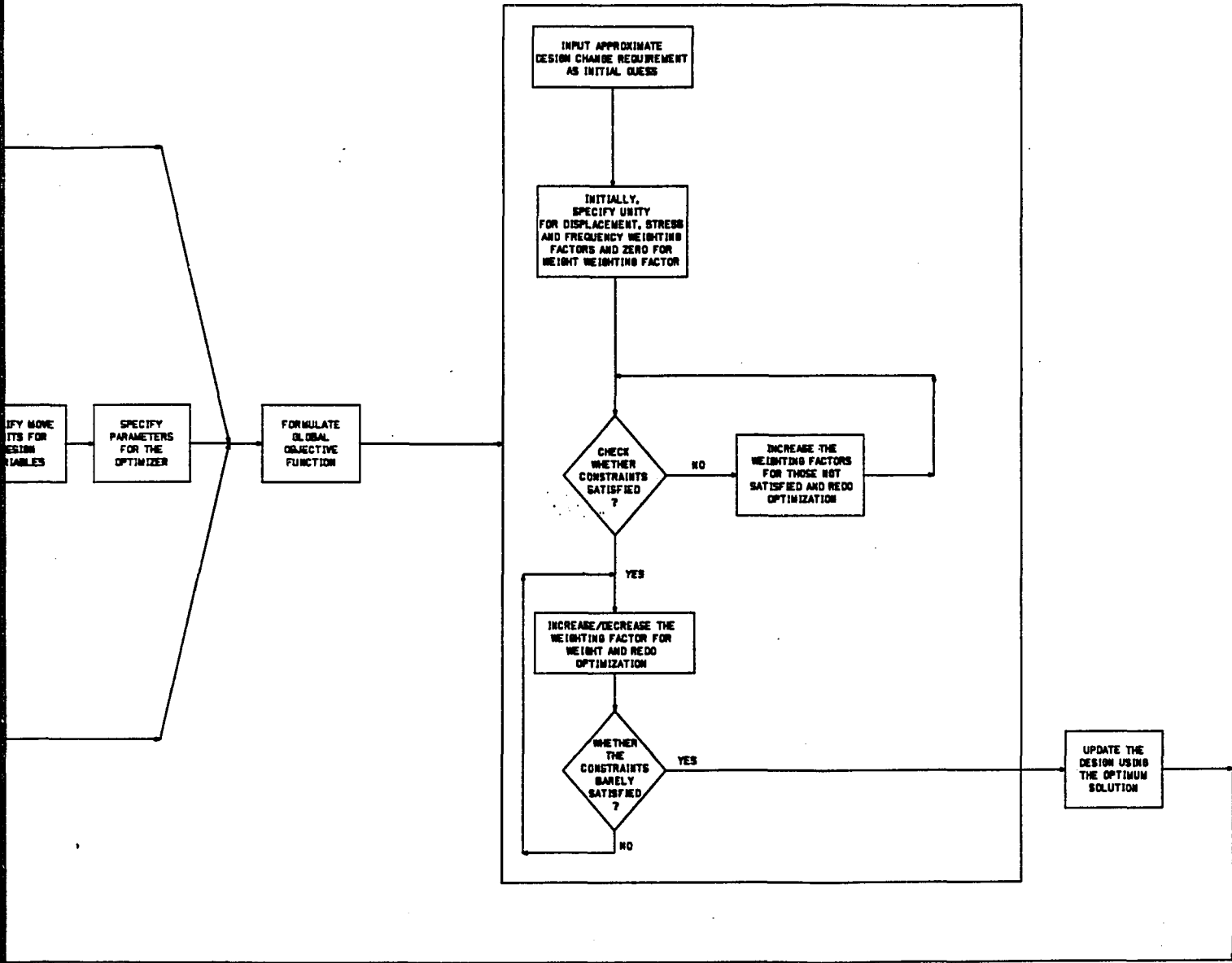


Figure 4.1: The flow chart of the o

PART OF THE OPTIMIZATION SCHEME



Flow chart of the optimization scheme.

4.1 Analysis Model .vs. Design Model

This thesis distinguishes between an analysis model and a design model. The analysis model is the finite element model which is used to determine the response of a system. This thesis uses MSC/NASTRAN, commercial finite element analysis software from the MacNeal-Schwendler Corporation, for the analysis. The design model, on the other hand, specifies the design variables, response constraints, design variable size constraints, and the objective. Design variables are defined by reference to the analysis model. That is, each design variable refers to a particular element property such as thickness, cross-sectional area. Design variable linking allows to relate one design variable with the properties of several elements. In shape optimization, a design variable may refer to a combination of grid locations.

4.2 Critical Responses

The finite element analysis of the initial design provides the free vibration response of the structure and the deflection and stresses for various static loads. The designer assesses these results to determine whether the responses are acceptable or not. If the response is not satisfactory, there is a need for design changes. The designer chooses key responses, here called critical responses, which are incorporated as constraints in the optimization for improvement. For example, either an unacceptable displacement at a particular node or an unacceptable von Mises stress in a specific element or an undesirable natural frequency could be a critical response. The critical response locations, i.e., particular nodes and elements, may change during the course of optimization. Therefore, the constraints have to be checked for criticality and added or dropped for the following iteration.

4.3 Weighting Factors

A linear combination of the individual response constraint functions is formulated to obtain the so called "global" objective function. An example is:

$$\begin{aligned} C = & P(\textit{CombinationCF}) + Q(\textit{DisplacementCF}) + R(\textit{StressCF}) \\ & + U(\textit{FrequencyCF}) + T(\textit{Weight - of - the - design}) \end{aligned}$$

Where CF indicates a constraint function. The weighting factors of the constraint functions $P, Q, R, U,$ and T dictate the relative importance of each constraint function. The choice of weighting factors best suited for the problem at hand is not usually obvious and requires several adjustments to determine an acceptable design. This thesis takes the view that this is the appropriate place for engineering judgement. Thus the procedure is set up so that the designer can choose the weighting factors interactively.

Note that it is not our goal to find weighting factors which yield small values of the objective function C . Rather, the designer chooses weighting factors and the software finds the values of the design variables which minimize C . The designer assesses the attributes of that design for its acceptability, and has the option of changing weighting factors to get a new design. In short, the designer is interactively determining an objective function whose minimum leads to a good design.

After the designer is satisfied that he has interactively determined the weighting factors for the desired cost function C^* , the structure is reanalyzed for the resulting set of design variables \mathbf{v}^* . In the examples to follow, we refer to the cycle of interactively choosing the weighting factors, finding \mathbf{v}^* , and resuming the finite element analysis as an iteration.

4.4 Approximate Problem

This thesis follows the concept of the approximate problem developed by Schmit and Miura[28] which makes it possible to obtain near-optimal designs with very few complete finite element analyses. The basic idea is to linearize the response functions using sensitivity information which forms the basis of a linear expansion about the original design. We expect this to be a reasonably good approximation over a fairly wide range of design variable values. MSC/NASTRAN software is used to obtain the sensitivity information. The explicit formulations replace the original implicit problem with an explicit approximate problem, and the approximate problem is then used in the optimization phase. Since evaluation of the linearized functions is not computationally demanding, this local optimization can be computed interactively.

If the designer wishes to continue, the linearization process is repeated about \mathbf{v}^* and new approximate optimization problem is solved. At each stage, move limits are imposed to insure that the structure is not changed so drastically that the linearization would obviously be invalid. The process of constraint linearization and optimization is repeated until no further design improvements can be found.

4.5 Optimizer

The goal of the optimizer is to find the design variable vector that minimizes the "global" objective function while satisfying the constraints through a numerical search procedure in the design space. A wide variety of optimization algorithms are available. They all generate a sequence of improved approximations to the minimum, each derived from the preceding approximation.

$$\mathbf{v}^{k+1} = \mathbf{v}^k + \alpha_k \mathbf{S}_k \text{ for } k = 0, 1, 2, \dots \quad (4.1)$$

where \mathbf{v}^{k+1} is the new or improved approximation in the k th stage,

\mathbf{v}^k is the approximation to the minimum obtained in the $(k - 1)$ th stage,

α_k is the step size,

\mathbf{S}_k is the search direction.

Thus, the optimization proceeds in two steps: first determine a "usable-feasible" search direction \mathbf{S}_k , and then perform a one-dimensional search in this direction to reduce the objective function as much as possible subject to the constraints. Some optimizers restrict the initial starting design to be feasible (satisfies all constraints). CONMIN (A Fortran program for Constrained Minimization)[27] is an optimizer used to minimize the "global" objective function. CONMIN implements the feasible direction algorithm of Zoutendijk[29]. This algorithm requires first derivatives of the objective function and constraints and has a linear rate of convergence. It is a robust algorithm and the initial, or baseline, design need not be feasible. One difficulty in a numerical optimization process is that a numerical search procedure can only seek a relative extremum, while multiple local extrema may exist due to the nature of either the objective function or the constraints or both. Thus in order to ensure that a global extremum is obtained, it may be useful to solve the problem more than once using different starting points.

4.6 Optimization Software

A general-purpose optimization package OPTIMIZE, was developed based on the concepts presented in this thesis. The program, which was written in FORTRAN, is currently running on a VAX 11/785 computer with VMS operating system. System flexibility and modularity were emphasized in the development. The software permits

the incorporation of additional objective functions and constraint functions. The whole system is based on a group of software modules, each performing a specific task, and allows linking to virtually any design problem which can be formulated mathematically as a nonlinear optimization problem. The organization of the optimization system is outlined in Figure 4.2.

THE ORGANIZATION OF THE OPTIMIZATION SYSTEM

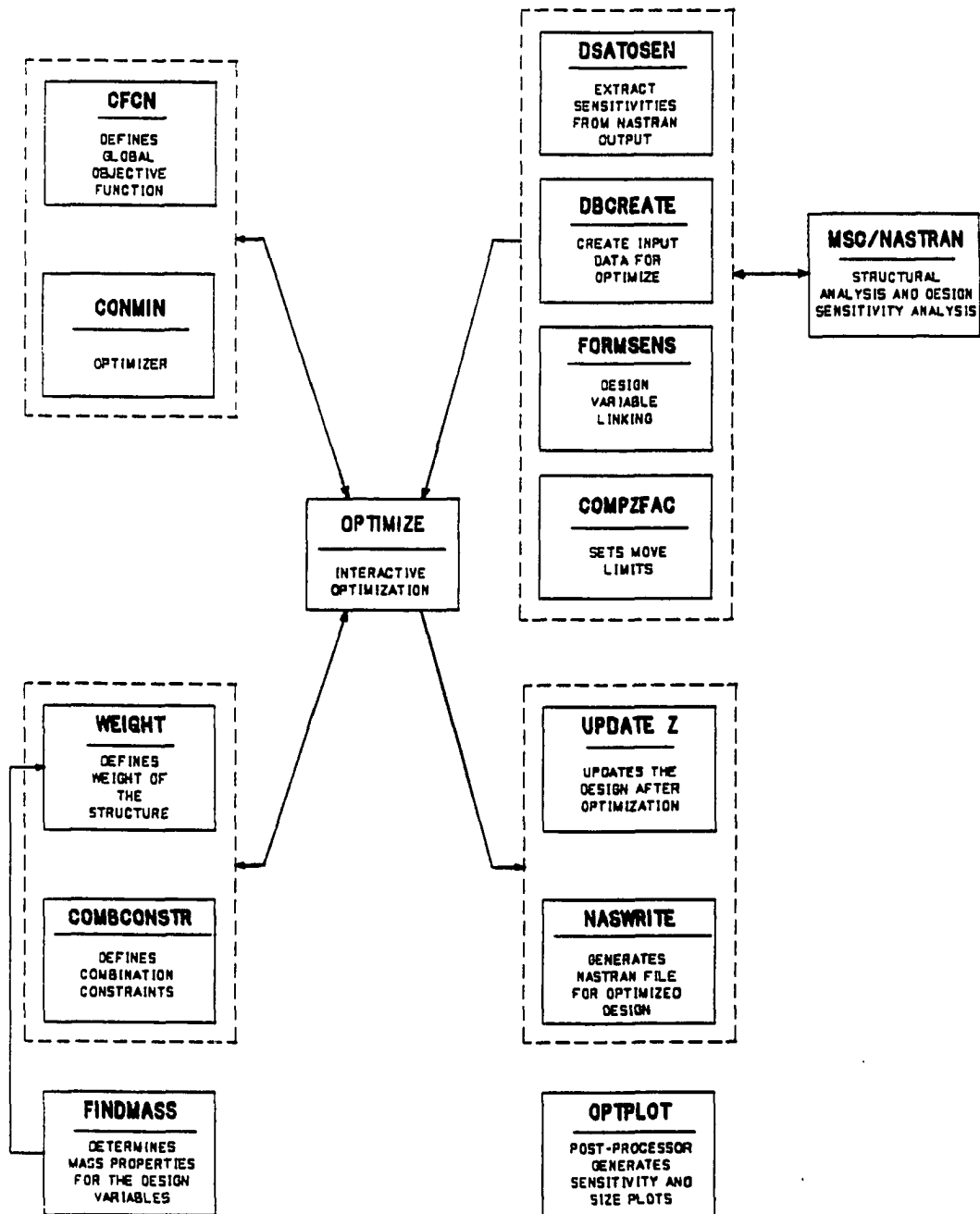


Figure 4.2: The organization of the optimization system.

The optimization software altogether consists of 12 modules. OPTIMIZE is the main program. It operates interactively by giving prompts and receiving input data which is then processed by interacting with other modules to minimize the given objective function. Through user-supplied modules WEIGHT and COMBCONST, the base program can be linked to formulate different design problems. The modules of the optimization software will be discussed below.

4.6.1 NASTRAN

NASTRAN is a general purpose structural analysis and design sensitivity analysis program. This software is used to determine the critical response quantities and their sensitivities to the changes in the design variables. A wide range of structural analysis problems can be treated with this program. These include:

- the determination of static displacements and stresses due to applied nodal loads, element pressures, and gravity fields in any directions, and
- the determination of natural frequencies and mode shapes.

Boundary conditions may be prescribed on any nodal displacement degree-of-freedom by fixing it or constraining it to be equal to other degrees-of-freedom.

Several elements such as thin shells, beams, trusses, solids, and lumped mass and stiffnesses are supported. NASTRAN supports sensitivity calculations based on the properties of these elements, i.e., shell element thickness, truss element section area, beam section area and moments of inertia, etc. Geometrically based design variables cannot be handled directly in NASTRAN and require special treatment for sensitivity analysis. Reference 30 presents a convenient procedure.

4.6.2 CFCN

CFCN is the key module for the OPTIMIZE software. It performs three basic tasks. First it evaluates the critical responses at a given search point in the design space using first order Taylor series approximations. Second it evaluates the appropriate constraint functions for displacements, stresses, frequencies, and combined displacement and design variables using the predicted values of the critical responses. Finally it computes the weighted sum of those individual constraint function as weighted by the designer to obtain the "global" objective function value. CONMIN calls this module during the search for optimum design variable vector.

4.6.3 WEIGHT

The WEIGHT module is an user-supplied subroutine which expresses weight as a function of the design variables. For example, the weight of a shell structure may be expressed as the product of the thickness which might be a design variable, the surface area of the structure, and the weight per unit volume of the material. Similarly, the weight of a rectangular beam structure can be expressed as the product of the design variables width and height, length of the beam, and the density of the material. The module evaluates the weight of the design for a given search point in the design space and returns the value to the CFCN module.

4.6.4 COMBCONST

The COMBCONST module is also an user-supplied subroutine which defines the combination constraints Y_i and their lower and upper limits (Y_i^l, Y_i^m). The combination constraints specify the limits either to a combination of displacement variables and design variables or to a combination of design variables only. A typical combination

constraint can be expressed as follows.

$$\begin{aligned}
 Y(2) &= V(5) + DISP(3) \\
 YLO(2) &= 1.0 \\
 YHI(2) &= 4.0
 \end{aligned}$$

Where $Y(2)$ is the combination constraint number 2, $V(5)$ is the design variable number 5, $DISP(3)$ is the displacement variable 3, $YLO(2)$ and $YHI(2)$ are the lower and upper limits for the constraint $Y(2)$. In this example, the constraint $Y(2)$ limits the sum of the displacement response at a node given by $DISP(3)$ and the size of the design variable 5 to the range (1.0,4.0) during the search for optimum. Similarly a combination constraint can set limits to the function of a group of design variables as follows.

$$\begin{aligned}
 Y(5) &= SQRT(V(2) * V(2) + V(4) * V(4)) \\
 YLO(5) &= 0.5 \\
 YHI(5) &= 2.5
 \end{aligned}$$

The combination constraints can contain any mathematical expression of the design variables and/or displacement variables. This module evaluates the combination constraint at the search point in the design space and returns the computed value and its user specified lower and upper limits to the CFCN module.

4.6.5 FINDMASS

The FINDMASS module provides the necessary information to prepare the WEIGHT subroutine. It computes the surface areas for shell element structures, lengths for beam and rod element structures, and outputs the results associated with finite element prop-

erty labels in a tabular form. The module takes the NASTRAN's finite element data file as input.

4.6.6 DSATOSEN

DSATOSEN is a utility module and not directly called by the OPTIMIZE program. This program process the sensitivity output file from NASTRAN and generates the design variable sensitivity data file (SENS.O) for OPTIMIZE.

4.6.7 DBCREATE

The DBCREATE module creates the necessary data files for OPTIMIZE program. It generates the following files as per the designer's directives.

PRSENT.O Contains the critical response values for the initial design.

DESIRED.O Contains the designer prescribed limits for displacement and stresses for different load conditions, and undesirable frequency bands.

INDDEP.O Contains the design variable linking data. Used to reduce the number of independent design variables and express the relative magnitude relationships among the design variables. FORMSENS module uses this file as input.

SIZES.O Contains the lower and upper limits on the size of the design variables.

OPTPAR.O Contains the control parameters for the optimizer CONMIN.

4.6.8 FORMSENS

The FORMSENS module implements the linking of design variables. A linear combination of a set of design variables is used to formulate the independent design variables. This provision is helpful for imposing realistic design constraints as well as

reducing the number of design variables. For example, the condition that the thickness t_1 of the molded rib on a surface structure made of composite material and the thickness t_2 of the base surface structure have to be maintained in certain proportion for manufacturing considerations, can be imposed easily using the FORMSENS module. The requirement, $\alpha_1 t_1 = \alpha_2 t_2$, can be imposed by linking t_1 and t_2 into an independent variable T . FORMSENS reads the design variable labels for t_1 and t_2 , the associated α values from INDDEP.O file and the associated sensitivities from SENS.O, and forms a linear combination of the sensitivities to obtain the effective sensitivity for the independent design variable. The design variables t_1 and t_2 are discarded and the independent variable T is used in the optimization process. Once the solution for T is obtained, the values for the design variables t_1 and t_2 can be determined using the α_1 and α_2 values.

4.6.9 COMPZFAC

The COMPZFAC module imposes move limits for the design variables. These limits confine the optimization search to a region in the design space where the approximate optimization problem is expected to be reasonably accurate. After obtaining the solution for the approximate problem, structural analysis and a sensitivity calculations are performed for the new design. Then a new approximate model is set up, bounded by another set of move limits. The lower and upper move limits are expressed as fractions of the size of the design variable. For example, a move limit range of (0.9, 1.1) for the 4th design variable means that the design change for the 4th design variable is limited to the range $0.9 * v_4^0 - 1.1 * v_4^0$ during the search for optimum. Where v_4^0 is the present size for the design variable 4.

4.6.10 UPDATEZ

The UPDATEZ module updates the present sizes of the design variables (SIZES.O) using the solution of the optimizer (SOLN.O), and generates a new SIZES.O file. The solution file SOLN.O contains the optimum design changes for the design variables as the fractions of the present sizes. A solution value of 0.1 for a variable means the optimum value is 1.1 times the present value.

4.6.11 NASWRITE

The NASWRITE module consists of subprograms which prepare NASTRAN data files for response and sensitivity analysis using updated values for the design variables(new SIZE.O). Subprograms PSHELLWRITE and PBARWRITE generate NASTRAN data files for shell and beam structures respectively.

4.6.12 OPTPLOT

The OPTPLOT module generates PDA/PATRAN(a commercial software for finite element pre-, post-processor marketed by PDA Engineering) neutral files from the sensitivity data(SENS.O) and the design variable size data(SIZES.O) for displaying the color-coded plots of the structure showing the sensitivities and final dimensions of the various portions of the structure. Different colors indicate different values for the design variables.

In order to illustrate the use of the OPTIMIZE optimization software system, two typical applications to the design of automotive structures subject to different kinds of constraints are presented in the next chapter.

5 EXAMPLES

This section presents two example problems which illustrate the optimization procedure. The first example concerns the block of an automotive engine which is redesigned to shift the natural frequencies while retaining minimum mass. The second is a far more complicated problem concerning the mounting of a heavy-duty truck's vertical exhaust pipe which is redesigned to satisfy the displacement, stress, and natural frequency requirements, again with minimum mass.

5.1 Engine Block Redesign

The block of an internal combustion engine is a geometrically complex component which is exposed to a variety of excitation forces such as pressures developed during combustion process and inertial imbalance forces generated by reciprocating components. The vibration of the block due to these excitations results in unwanted engine noise. The engine noise can be greatly reduced by suppressing the basic natural vibration of the block in the dominant excitation frequency range through structural modifications.

The engine employed for this analysis was a 1900 cc watercooled in-line 4-cylinder gasoline engine for automobile use. The finite element method was used for analyzing the dynamic structural behavior of the block. The finite element mesh consisted of 1413 nodes and 1420 shell elements. The material is cast iron.

Figure 5.1 presents the finite element model of the initial design. NASTRAN was

Table 5.1: Natural frequencies of the initial engine block design.

Mode No.	Natural frequency(Hz.)
1	3.274097E+02
2	3.305477E+02
3	3.409858E+02
4	3.453373E+02
5	4.207336E+02
6	6.457792E+02
7	8.147952E+02
8	8.589611E+02
9	9.194916E+02
10	9.512925E+02
11	9.536907E+02
12	1.125018E+03
13	1.133627E+03
14	1.141519E+03
15	1.226497E+03
16	1.236966E+03
17	1.291474E+03
18	1.297739E+03
19	1.328297E+03
20	1.354187E+03

used to obtain the eigenvalues in a range up to 1400 Hz. Table 5.1 lists 20 modes in the chosen frequency range. The block weighed 24.62 Kg.

In the literature[31], it was indicated for a similar engine that the frequency range of 700 Hz through 900 Hz has the highest percent contribution to the engine noise. Therefore, in this example, elimination of natural frequencies in the range 700 Hz - 900 Hz with the fundamental frequency not less than 300 Hz was the design goal. The lower limit 300 Hz on the frequency forces the fundamental frequency of the improved design to be in the vicinity of the initial design's fundamental frequency of 327 Hz and

Table 5.2: Lower and upper size limits for the design variables.

Var. No.	Present Size(mm.)	Lower Limit	Upper Limit
1	4.250	2.00	8.00
2	4.250	2.00	8.00
3	4.250	2.00	8.00
4	4.250	2.00	8.00
5	8.000	4.00	15.0
6	8.000	4.00	15.0
7	8.000	4.00	15.0
8	8.000	4.00	15.0
9	10.00	4.00	15.0
10	10.00	4.00	15.0
11	23.00	10.0	30.0
12	4.000	2.000	8.00

avoids resonance from any exciations below 300 Hz.

Table 5.1 indicates there are two frequencies in the undesirable range: 815 Hz (mode 7) and 859 Hz (mode 8). The corresponding mode shapes are shown in Figures 5.2 and 5.3. The frames indicate that these are modes of lateral bending of the front and back faces the block.

For this example twelve design variables were selected. Each one delineates a region of the block. Figure 5.4 shows the regions for different design variables. Table 5.2 presents the thickness of these regions for the initial design and the allowable thickness limits.

All the twenty natural modes were included in the optimization process to make sure no modes were forced into the undesirable frequency band while shifting the modes 7 and 8 out of the undesirable frequency band. The eigenvalues, which hold implicit relationships with design variables, were approximated using explicit linear approxima-

tions.

$$\lambda_i(\mathbf{v}) = \lambda_i(\mathbf{v}_0) + \sum_{j=1}^t \frac{d\lambda}{dv_{0j}}(v_j - v_{0j}) \quad (5.1)$$

In this approximation, the eigenvalues are linear in the design variables v_j and their derivatives (sensitivities) are assumed constant: $\frac{d\lambda}{dv_i} = \frac{d\lambda}{dv_{0i}}$. The sensitivities for all the twenty eigenvalues were computed using NASTRAN. The spatial variation of the sensitivities for modes 7 and 8 are shown in Figure 5.5 and 5.6 respectively.

The design problem is the selection of suitable thickness for the engine block to shift the natural frequencies out of the undesirable frequency band and to permit weight reduction. These competing objectives lead to a vector optimization problem with two objective functions: minimal weight $W(\mathbf{v})$ and frequency constraint function $F(\mathbf{f})$. In this example, there are two undesirable frequency bands which are in the ranges 0 - 300 Hz and 700 - 900 Hz. Therefore, the frequency constraint function is composed of two component functions F_1 and F_2 .

$$F_1(\mathbf{f}) = \begin{cases} \sum_{i=1}^m [1 - \cos(2\pi f_i/300.)] & \text{for } -10.0Hz. \leq f_i \leq 300.0Hz. \\ 0 & \text{for } f_i \notin (0.0, 300.0)Hz. \end{cases} \quad (5.2)$$

$$F_2(\mathbf{f}) = \begin{cases} \sum_{i=1}^m [1 - \cos(2\pi(f_i - 700.)/200.)] & \text{for } 700.0Hz. \leq f_i \leq 900.0Hz. \\ 0 & \text{for } f_i \notin (700., 900.)Hz. \end{cases} \quad (5.3)$$

Where m is the number of modes in the finite element model, \mathbf{f} is the modified frequency vector and f_i is the i th modified natural frequency. Figure 5.7 shows the frequency constraint function for this example.

Following equation (3.1), we obtain a scalar problem by formulating the global objective function which is the weighted sum of these two objective functions.

$$C(W, \mathbf{f}, \mathbf{v}) = U [F_1(\mathbf{f}) + F_2(\mathbf{f})] + TW(\mathbf{v}) \quad (5.4)$$

The weighting factors U and T are determined interactively while minimizing $C(W, f, v)$ subject to side limits on the design variables to obtain an optimal solution. The optimization task is accomplished in two steps: a frequency-modification step and a weight-minimization step. In the frequency-modification step, the weighting factor for frequency constraint function U is set to a constant and weighting factor for weight function T to zero to shift the natural frequencies out of the undesirable bands. In the weight-minimization step, the weighting factor for weight function is gradually increased to reduce the weight while maintaining the frequency spectrum. Typical optimization output is presented in Appendix.

Since the linear approximations to the eigenvalues are valid only for a limited range, move limits are placed on the range of design change admissible in any iteration. The move limits for the next iteration are set based on how well the linear approximations predicted the response in the present iteration. In this example, 30% change was allowed in the first iteration. In the second the change was reduced to 20%. In the subsequent iterations, the move limit was further reduced to only 10%.

The optimization process took 5 iterations. Figure 5.8 shows the variation of the block weight with the number of iterations. The final design weighs 18.44 Kg, 25% less than the initial design. The history of natural frequencies and the block thicknesses for the 5 iterations are shown in Tables 5.3 and 5.4 respectively. The natural frequencies of the final design do not lie in the undesirable frequency bands.

Table 5.3: Thicknesses for different optimization iterations.

Var. No.	INIT.	ITR-1	ITR-2	ITR-3	ITR-4
1	4.250	4.4237	4.6954	4.20120	4.16640
2	4.250	5.5250	5.0512	5.48070	5.57120
3	4.250	5.5250	5.0369	5.47760	5.48540
4	4.250	4.4599	4.7131	4.36660	4.24520
5	8.000	8.3478	9.3461	8.37340	8.04760
6	8.000	7.2284	6.1380	5.53990	5.49980
7	8.000	7.2302	6.1371	5.53290	5.46750
8	8.000	8.3619	9.3490	8.38290	8.03380
9	10.00	7.0000	5.6000	4.76000	4.28400
10	10.00	8.2903	7.6126	7.40000	7.83190
11	23.00	16.1000	12.8800	10.94800	10.00000
12	4.000	2.80000	2.2400	2.00010	2.00000

Table 5.4: Natural frequencies for different optimization iterations.

Mode No.	INITIAL	ITERATION 1	ITERATION 2	ITERATION 3	ITERATION 4
1	3.274097E+02	3.049393E+02	3.042326E+02	3.006863E+02	3.202546E+02
2	3.305477E+02	3.111058E+02	3.077774E+02	3.066557E+02	3.266078E+02
3	3.409858E+02	3.372145E+02	3.265911E+02	3.340571E+02	3.569066E+02
4	3.453373E+02	3.414173E+02	3.345806E+02	3.397365E+02	3.631082E+02
5	4.207336E+02	4.032441E+02	3.874937E+02	3.918759E+02	4.179890E+02
6	6.457792E+02	6.819006E+02	6.796697E+02	6.816149E+02	6.858755E+02
7	8.147952E+02	8.880566E+02	8.982814E+02	8.980705E+02	9.016401E+02
8	8.589611E+02	1.006229E+03	9.377675E+02	9.302898E+02	9.321375E+02
9	9.194916E+02	1.057986E+03	9.815551E+02	9.374390E+02	9.364892E+02
10	9.512925E+02	1.072542E+03	9.891832E+02	9.406150E+02	9.387814E+02
11	9.536907E+02	1.074799E+03	1.018268E+03	9.726483E+02	9.724603E+02
12	1.125018E+03	1.081950E+03	1.021502E+03	9.996768E+02	9.963218E+02
13	1.133627E+03	1.087908E+03	1.033367E+03	1.008212E+03	9.986893E+02
14	1.141519E+03	1.110399E+03	1.037882E+03	1.011534E+03	1.003139E+03
15	1.226497E+03	1.170272E+03	1.091895E+03	1.068974E+03	1.066552E+03
16	1.236966E+03	1.234622E+03	1.128939E+03	1.091303E+03	1.086475E+03
17	1.291474E+03	1.255379E+03	1.202084E+03	1.143626E+03	1.099165E+03
18	1.297739E+03	1.268252E+03	1.246851E+03	1.210715E+03	1.178752E+03

FINITE ELEMENT MODEL OF THE ENGINE BLOCK

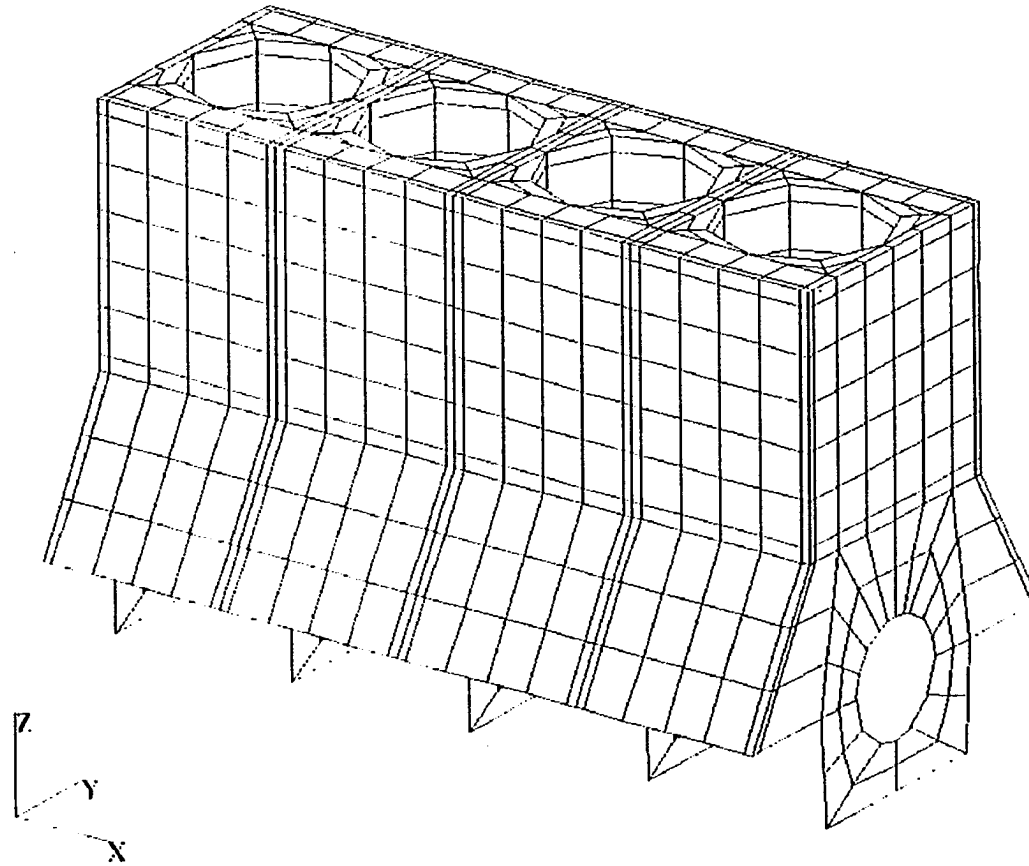
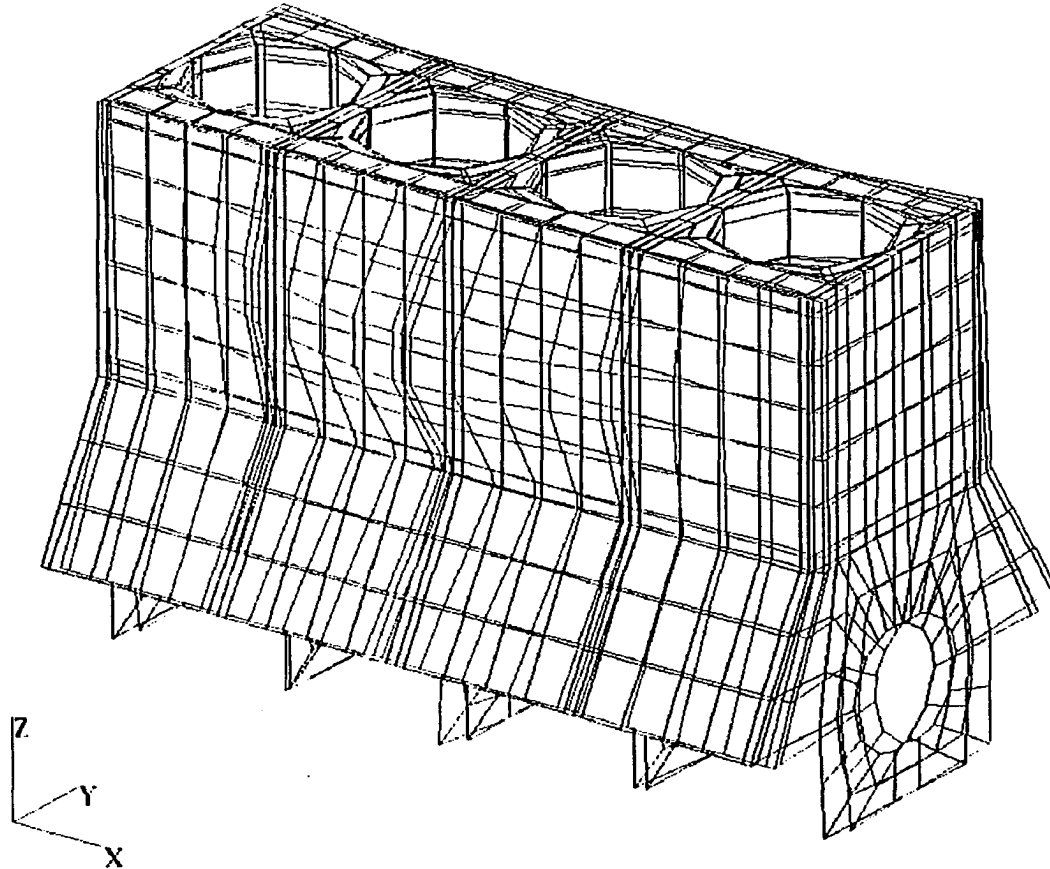


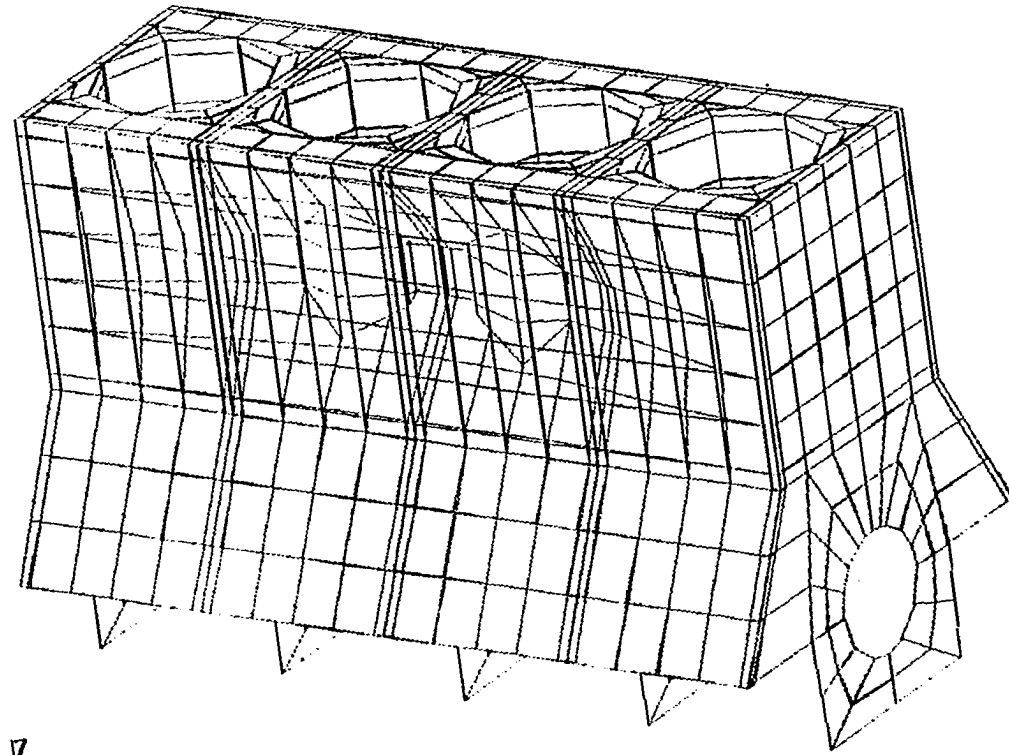
Figure 5.1: Finite element model of the engine block.



MODE SHAPE OF THE ENGINE BLOCK

MODE 7 @ 815 Hz.

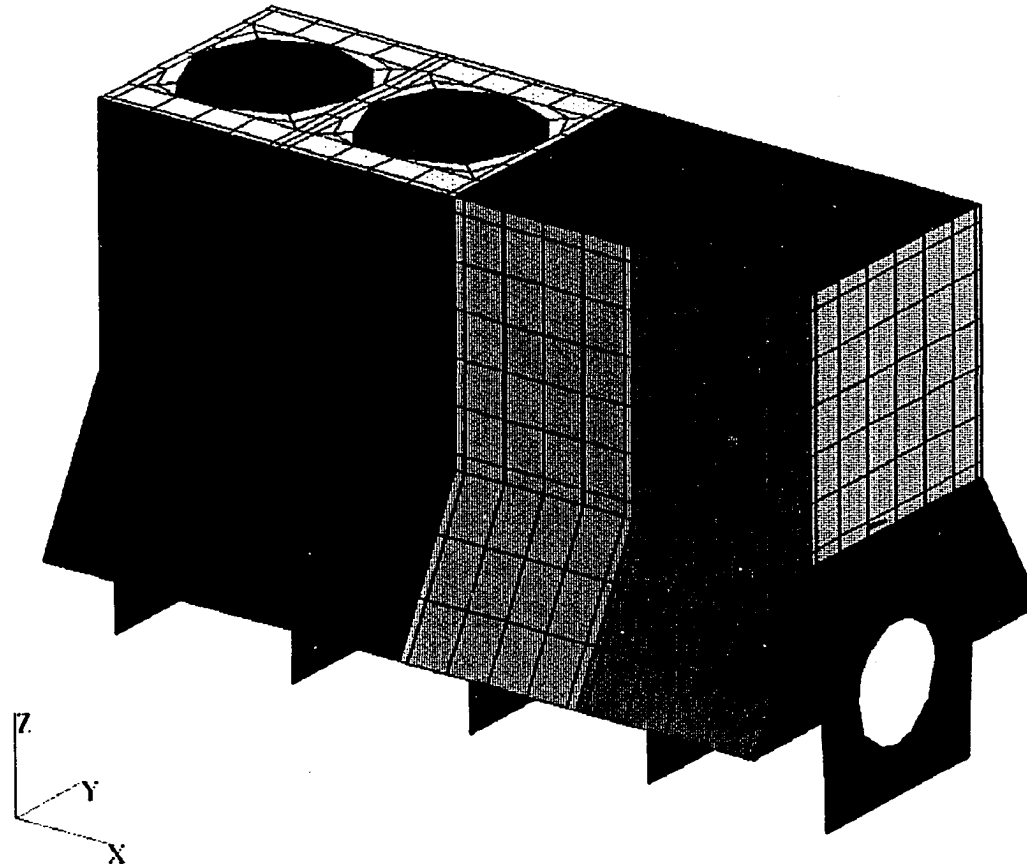
Figure 5.2: Mode shape of the engine block (Mode 7 - 815 Hz.).



MODE SHAPE OF THE ENGINE BLOCK
MODE 8 @ 859 Hz.

Figure 5.3: Mode shape of the engine block (Mode 8 - 859 Hz.).

DESIGN VARIABLES FOR THE ENGINE BLOCK DESIGN



DESIGN VARIABLES FOR THE OPTIMIZATION
REGIONS ARE COLOR CODED ACCORDING TO
THE DESIGN VARIABLE NUMBER.

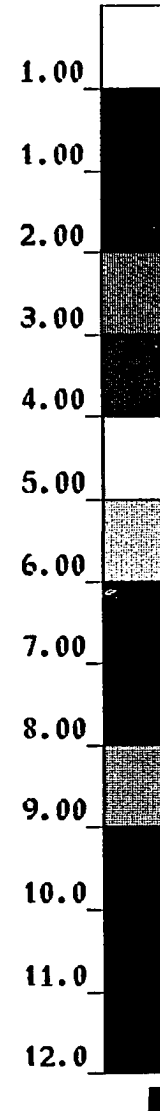


Figure 5.4: Design variables for the engine block design.

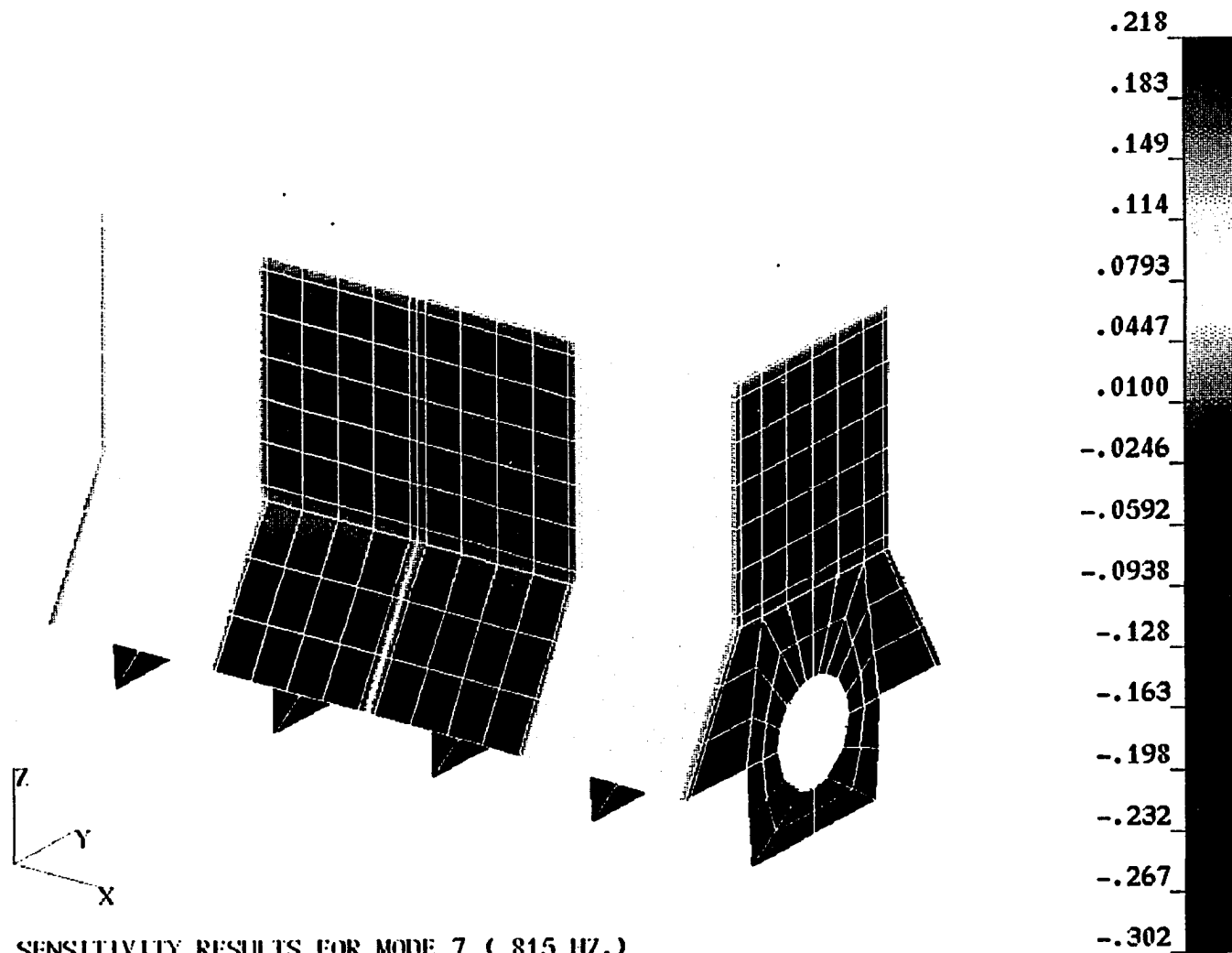


Figure 5.5: Sensitivity plot for the engine block: Mode 7.

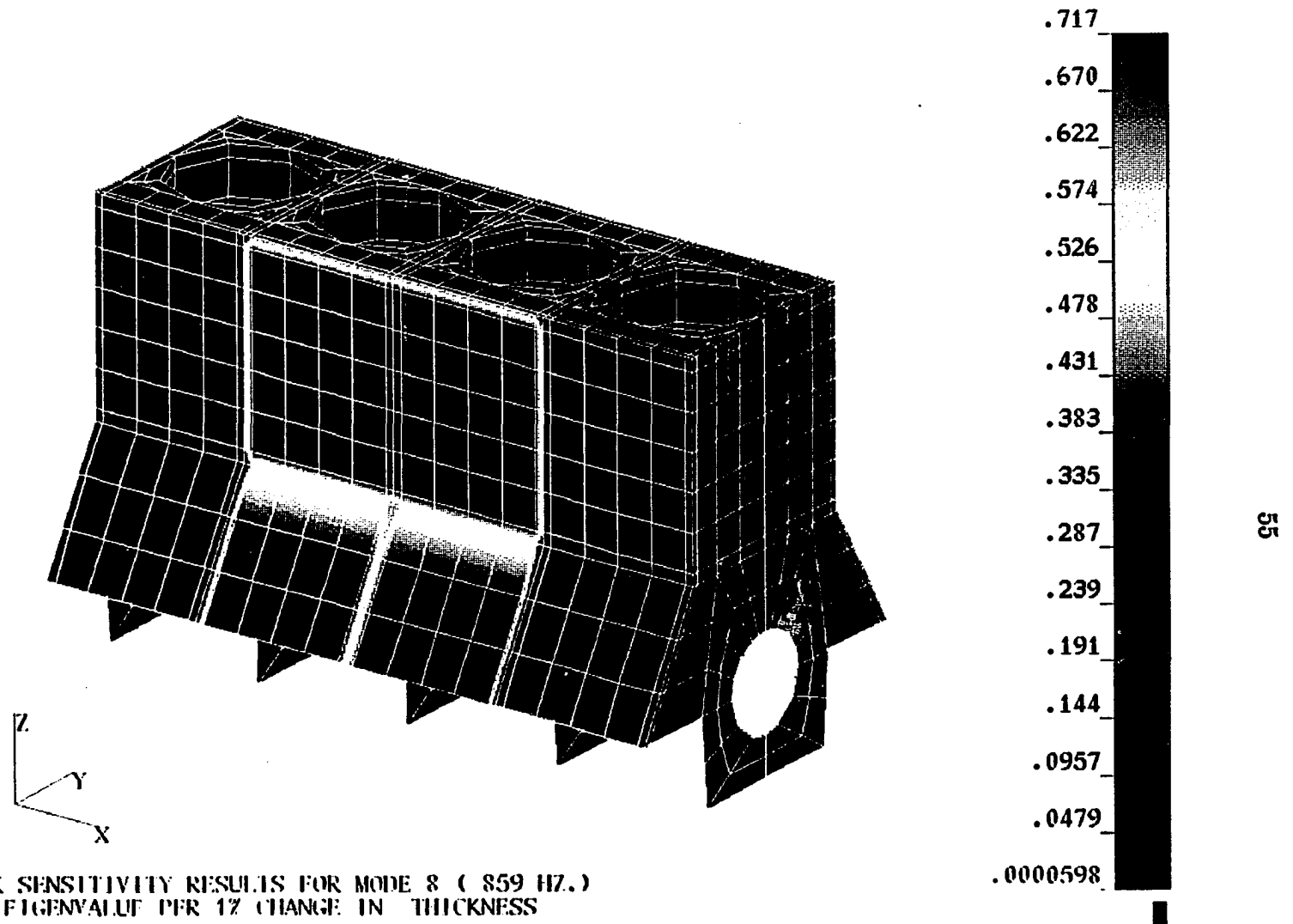


Figure 5.6: Sensitivity plot for the engine block: Mode 8.

FREQUENCY CONSTRAINT FUNCTION FOR ENGINE BLOCK

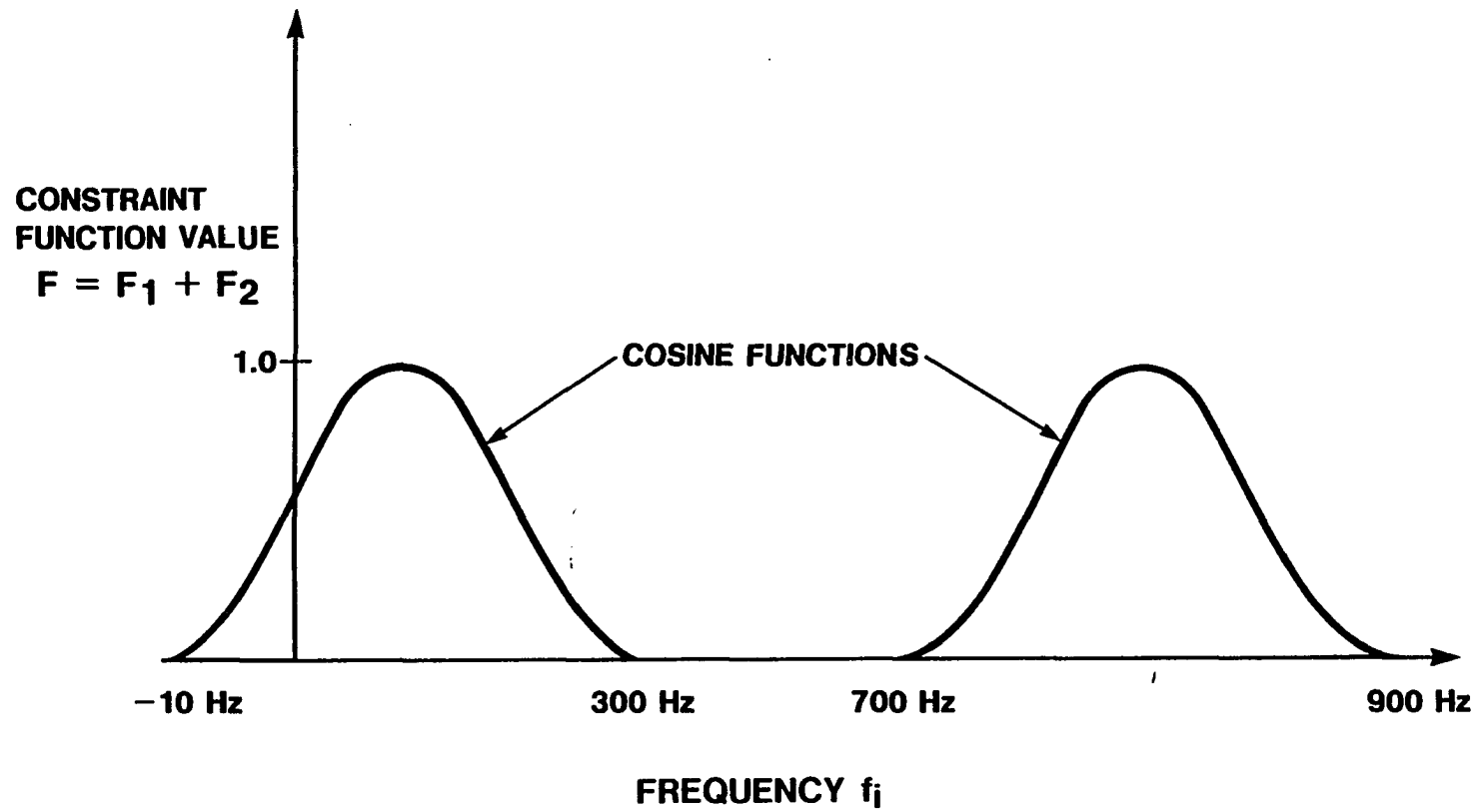


Figure 5.7: Frequency constraint function for the engine block.

ENGINE BLOCK
WEIGHT VARIATION WITH NUMBER OF ITERATIONS

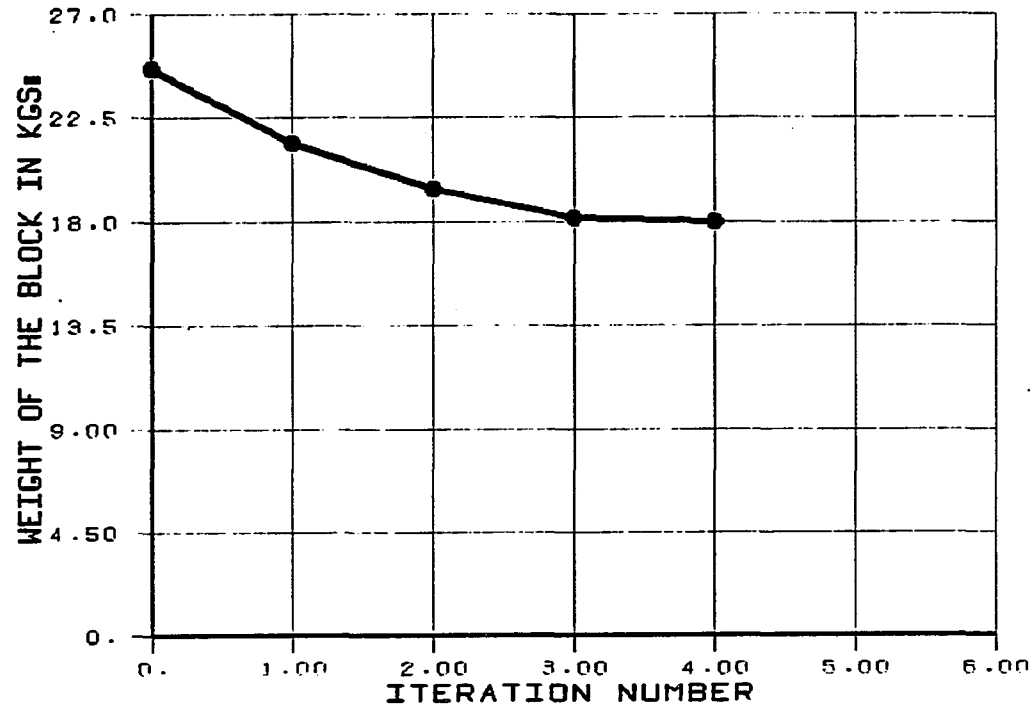


Figure 5.8: The variation of engine block weight with the number of iterations.

5.2 Exhaust Pipe Mounting

The design of a vertical exhaust pipe mounting system for a heavy duty truck is presented in this example. Figures 5.9 and 5.10 show the isometric and top views of the mounting arrangements. The material is steel. The exhaust pipe is mounted to the web of the framerail between the cab and the van by two rectangular tubes. The outboard ends of the tubes are welded to brackets which are fastened to the exhaust pipe. The inboard ends of the tubes are welded to a single bracket which is bolted to the framerail. A flat plate gusset is welded to the lower tube's bottom surface to reinforce the tube in longitudinal direction. (Refer Figures 5.9 and 5.10 for the nomenclature.)

The finite element model of the mounting structure was developed using NAS-TRAN. The supporting tubes were modeled with 8 beam(CBAR) elements each. The framerail bracket was modeled using 72 shell(CQUAD4) elements. The gusset was modelled using two concentrated mass(CONM2) elements. The exhaust pipe was modelled using 20 beam(CBEAM) elements. The finite element model, which is shown in Figure 5.11, has 816 displacement degrees-of-freedom. There are seven design variables in this example.

1. Upper Tube Width W_1
2. Upper Tube Height H_1
3. Upper Tube Thickness t_1
4. Lower Tube Width W_2
5. Lower Tube Height H_2
6. Lower Tube Thickness t_2
7. Framerail Bracket Thickness T

Table 5.5: Structural characteristics of the initial design

A) Design Variables	
Upper Tube	
1) Width W_1	2.000 inch
2) Height H_1	2.000 inch
3) Thickness t_1	0.083 inch
Lower Tube	
4) Width W_2	2.000 inch
5) Height H_2	2.000 inch
6) Thickness t_2	0.083 inch
Framerrail Bracket	
7) Thickness T	0.250 inch
B) Total weight	52.80 lb
C) Natural Frequencies	
Mode 1(Longitudinal)	9.730 Hz.
Mode 2(Lateral)	10.57 Hz.
D) Static Displacement Due	
To 6g Longitudinal Gravity Load	
@ Node 18	0.472 inch
@ Node 120	1.111 inch
E) Maximum Stress Due To	
To 10g Lateral Gravity Load	
Element 1	20,530 psi.

The initial values of the design variables and some of the computed results are summarized in Table 5.5. Mode shapes are shown in Figures 5.12 and 5.13.

A prototype of the initial design was made for testing. In over the road testing, the maximum measured accelerations at the upper exhaust pipe mounting bracket were 6g and 10g in longitudinal and lateral directions respectively. The lower tube of the prototype cracked during testing at its upper edge near the weld to the framerrail bracket(Element 1 in the model). This failure was the result of lateral vibration.

The failure of the prototype indicated that attention was needed in the area of the

model designated by Element 1. The failure can be prevented in two ways, namely, to increase the fundamental longitudinal and lateral frequencies from their current values of about 10 Hz. to at least 15 Hz. This increase in natural frequencies would help isolate the system from typical excitation frequencies. Secondly, the stress in the element under a 10g lateral gravity load is limited to about 15,000 psi.

$$STR_1 < 15,000psi \quad (5.5)$$

Where STR_1 is the maximum stress in Element 1.

There were also several displacement constraints to consider based on anticipated use. For example, the displacement at the top of the exhaust pipe (FE node 120) is limited by the 3.0 inch clearance between the exhaust pipe and van body. The displacement of the upper exhaust mounting bracket (FE node 18) is limited by the 1.0 inch clearance between the upper tube and the cab. These constraint equations are

$$DLN_{120} < 3.00inch \quad (5.6)$$

$$DLN_{18} < 1.00inch \quad (5.7)$$

Where DLN_{18} and DLN_{120} are the displacements at the top of the exhaust pipe mounting bracket and at the upper exhaust pipe mounting bracket respectively. Our design goal was to meet the constraint under a 6g longitudinal load.

The supporting tubes lie in the narrow gap between the cab and the van body. Therefore, the width of the upper tube is limited by the size of the gap and the longitudinal displacement of the upper tube to avoid hitting the van.

$$W_1 + DLN_{18} < 3.00inch \quad (5.8)$$

The tubes are welded to the framerail bracket at an angle; therefore, the total projected height of the upper and lower tubes on to the framerail bracket cannot

Table 5.6: Lower and upper size limits for the design variables.

Var no.	Design Variable	Lower Limit(in.)	Upper Limit(in.)
1.	Upper Tube - Width W_1	1.000	3.000
2.	Upper Tube - Height H_1	1.000	3.000
3.	Upper Tube - Thickness t_1	0.050	0.250
4.	Lower Tube - Width W_2	1.000	3.000
5.	Lower Tube - Height H_2	1.000	3.000
6.	Lower Tube - Thickness t_2	0.050	0.250
7.	Framerail Bracket Thickness T	0.050	0.250

exceed the height of the bracket.

$$H_1 / \cos 60 + H_2 / \cos 35 < 9.75inch \quad (5.9)$$

The angles 60 deg and 35 deg are the inclination angles of the upper and lower tubes with respect to the framerail bracket and 9.75 inch is the available height of the framerail bracket. Finally, common practice and experience led to the size constraints shown in Table 5.6.

The global objective function was formulated along the lines of the previous example. In this example, we have displacement and stress constraint functions in addition to the frequency constraint function. The design was optimized by interactively choosing the weight of the constraint functions. The optimization task was achieved in two steps: a system performance-improvement step and a weight-minimization step. The weighting factors were adjusted in the initial stages until the desired performance was reached. Then the weighting factor for weight was gradually increased to reduce the weight while the performance of the structure was reasonably maintained. To compensate for any performance drift which may occur during the weight-minimization mode, an occasional step in the performance improvement mode was introduced to return

Table 5.7: Structural characteristics of the optimized design

A) Design Variables	
Upper Tube	
1) Width W_1	2.770 inch
2) Height H_1	2.090 inch
3) Thickness t_1	0.088 inch
Lower Tube	
4) Width W_2	2.720 inch
5) Height H_2	2.910 inch
6) Thickness t_2	0.128 inch
Framerrail Bracket	
7) Thickness T	0.499 inch
B) Total weight	65.22 lb
C) Natural Frequencies	
Mode 1(Longitudinal)	15.29 Hz.
Mode 2(Lateral)	21.87 Hz.
D) Static Displacement Due	
To 6g Longitudinal Gravity Load	
@ Node 18	0.160 inch
@ Node 120	0.438 inch
E) Maximum Stress Due To	
To 10g Lateral Gravity Load	
Element 1	8,720 psi.

the performance to the desired values. That procedure led to the results summarized in Table 5.7. The process took one iteration. The limit on the frequencies had significant effect on weight as compared to the limits on stress and deflections. The finite element analysis with the design variables set to the Table 5.7 values verified that linear sensitivities were a valid approximation and Table 5.7 indicated the desired minimum.

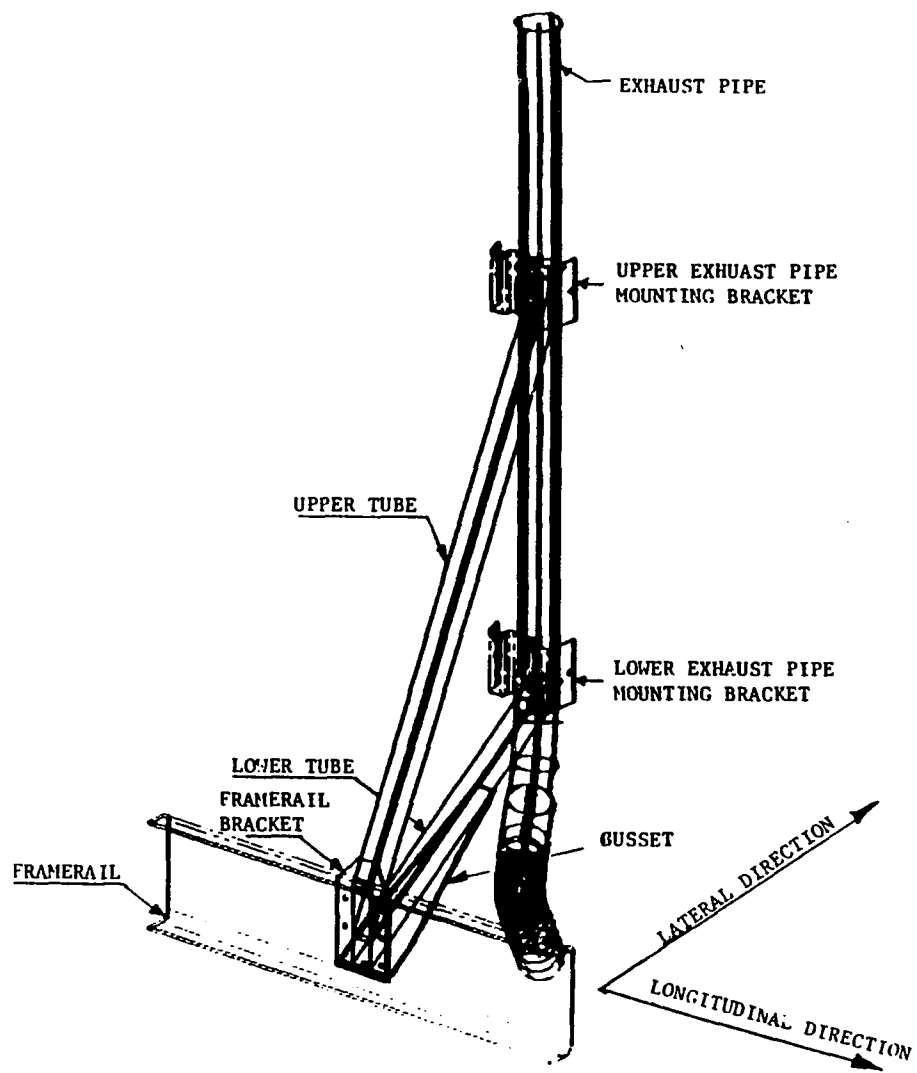


Figure 5.9: Exhaust pipe mounting arrangement: Isometric view.

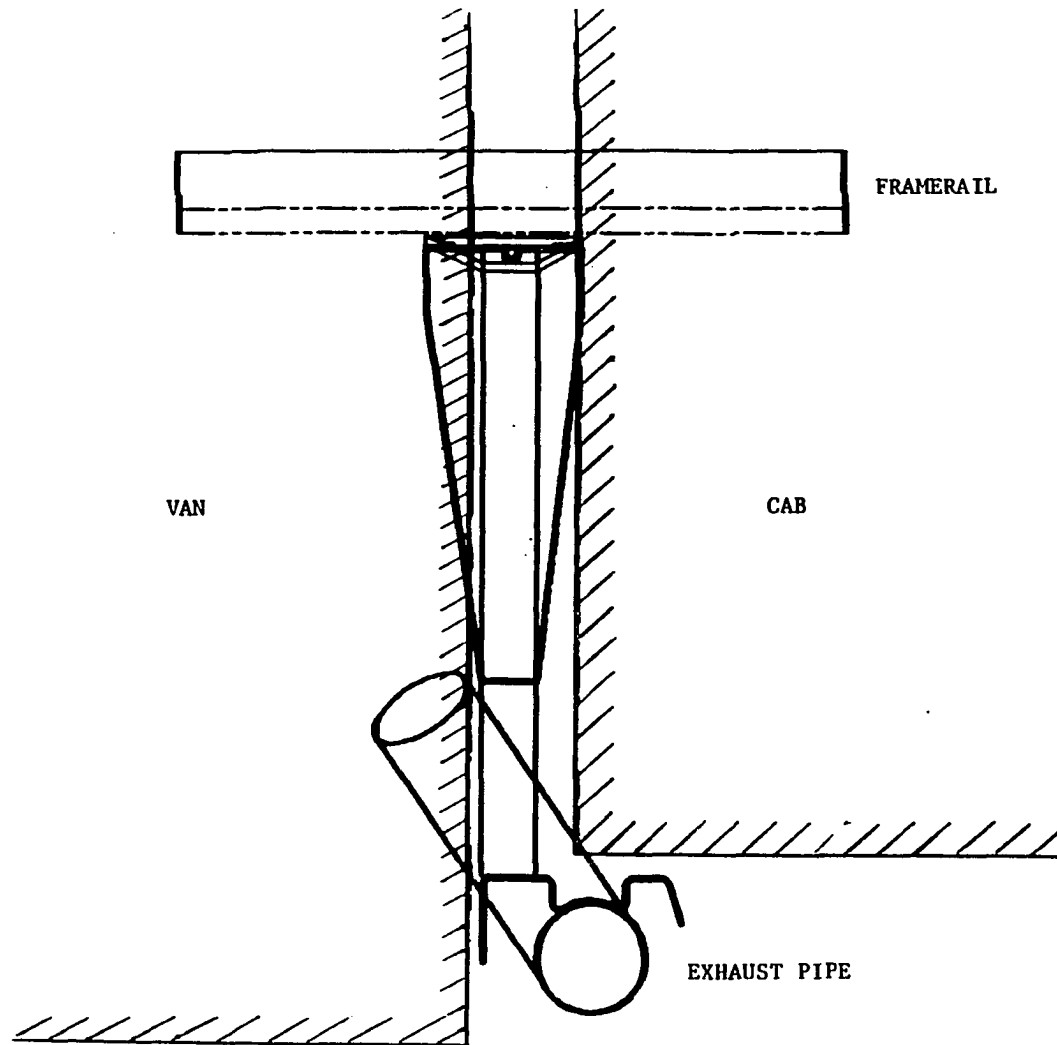


Figure 5.10: Exhaust pipe mounting arrangement: Top view.

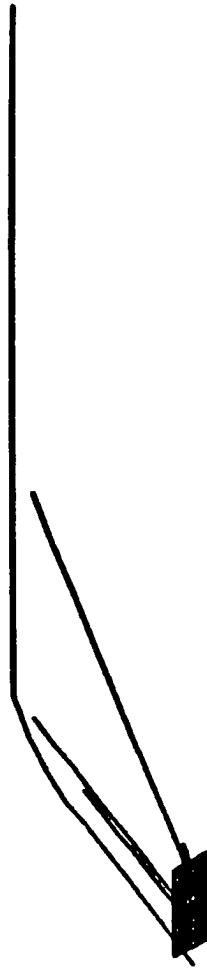
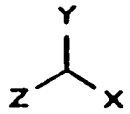


Figure 5.11: Finite element model of the exhaust pipe mounting structure.

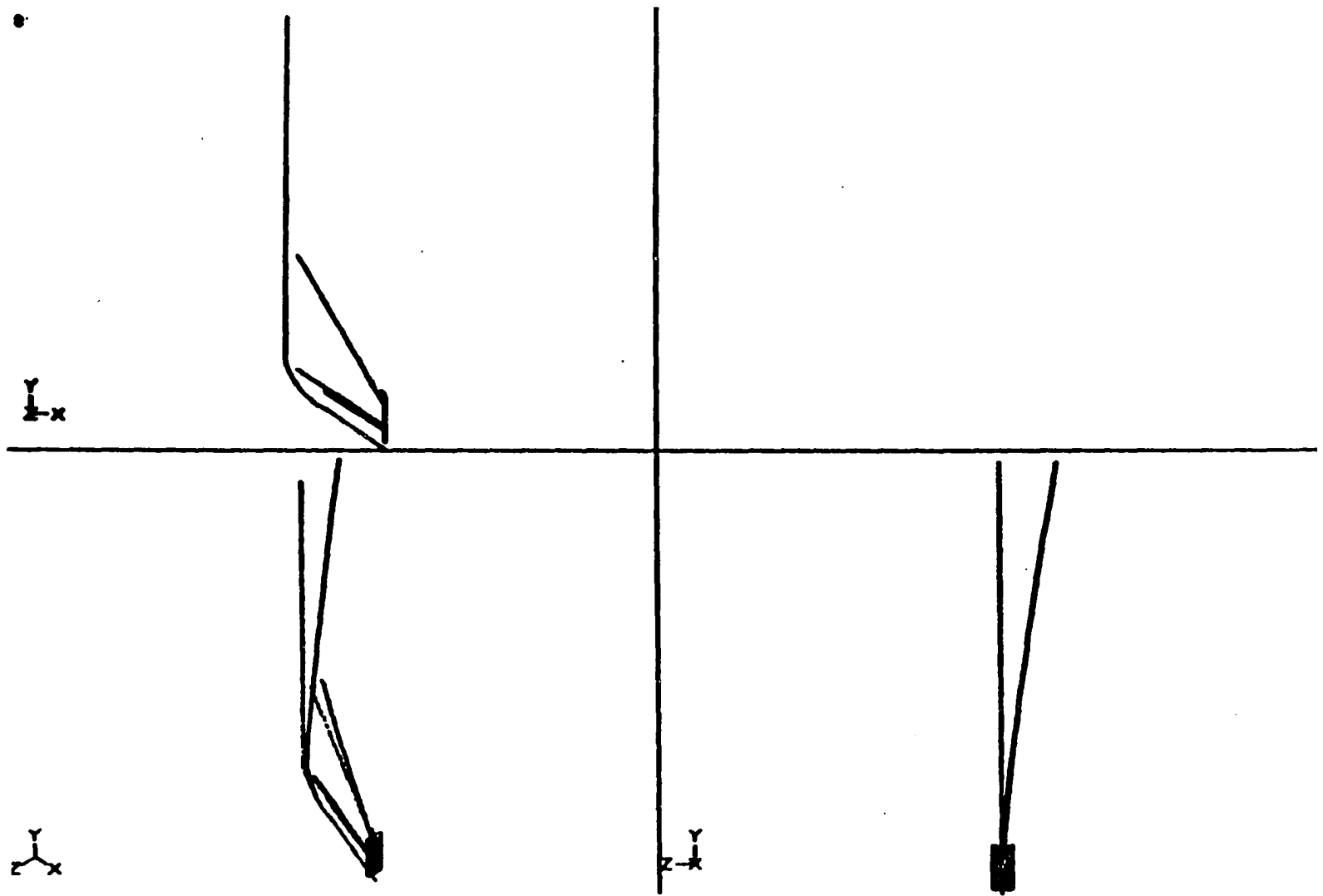


Figure 5.12: Mode shape of the exhaust pipe mounting: 9.73 Hz.(Longitudinal).

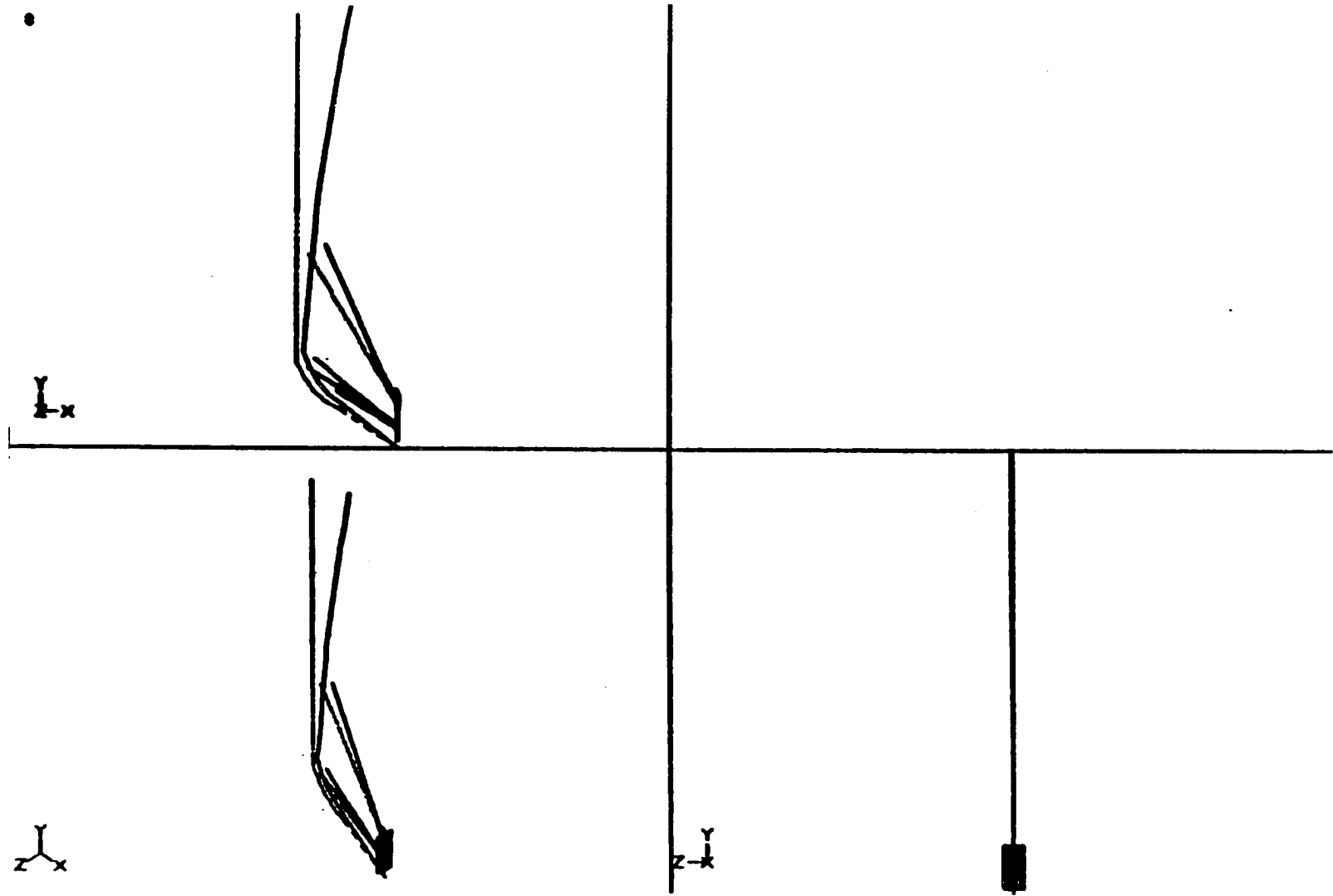


Figure 5.13: Mode shape of the exhaust pipe mounting: 10.57 Hz.(Lateral).

6 CONCLUSIONS

In the design of mechanical systems, the designer is often faced with the problem of satisfying a number of competing requirements, including light weight and limits on displacements, stresses, and natural frequencies. This usually leads to an optimization problem with weight as the objective and the performance requirements as constraints. It may be desirable, in addition to minimizing weight, to minimize or maximize other performance measures rather than enforcing performance limits.

This thesis recast the optimization problem as the minimization of multiple objectives, including performance indices and weight, subject to restrictions placed on the size of the design variables. The thesis also developed an interactive optimization procedure to enable design engineers to bring their skill into play during the optimization process.

The procedure transforms the multi-objective problem into a single objective problem by taking the weighted sum of the performance indices and the structure's weight. The weighting factors reflect the relative importance to the designer of the various conflicting objectives. The choice of the weighting factors best suited for the problem is not generally obvious and may require several adjustments before leading to an acceptable design. Thus the procedure is set up so that the designer can choose the weighting factors interactively.

Optimization software was developed to provide designers with a decision making tool which is easy to use and provides useful information from which designer can confi-

dently proceed. Since the method uses the NASTRAN for the response and sensitivity calculations, it is applicable to almost any structure which can be modelled using finite elements.

Two examples illustrate the power of the technique. The first considers the redesign of an automotive engine block where the challenge was to find a low-weight design which has no natural frequencies in undesirable frequency bands. This problem illustrated the trade-offs between the weight and the natural frequencies, and it demonstrated the interactive process wherein the designer found a combination of the twelve design variables that met performance requirements with a very low weight structure. The second example concerned the redesign of the mounting structure of a heavy-duty truck's exhaust pipe. The structural optimization was stated as finding a low weight structure which satisfied limits on maximum displacements and stresses, and shifted the natural frequencies out of an undesirable band. Again, the design engineer interactively found a very nice solution.

In summary, this thesis presents an approach to solve multiobjective optimization problems. The approach allows the designer to choose the best design based on his view of trade-offs among the various objectives.

A key part of the method is the interactive role played by the design engineer, a role made possible through sensitivity analysis. The efficiency of the method therefore depends on the range of validity of the approximations underpinning the sensitivity. Thus future work should include applications of this method with more accurate response-to-design variable relationships to make the procedure more efficient computationally. A suggestion is to develop an adaptive move-limit strategy using the the optimal solution found at the end of the each iteration and the accuracy of the response prediction for that iteration. It will also be useful to incorporate graphics interface to the OPTIMIZE software package so that the designer can plot trade-off curves among the objectives.

Additional improvements might include the use of the higher order approximations along a search direction indicated by linear sensitivity.

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8 APPENDIX

8.1 Sample Output From The OPTIMIZE Program

8.1.1 Frequency-modification step

\$ run optimize

Enter the initial guess for these variables

0.0001

RESPONSE AT THE INITIAL GUESS:

FREQUENCIES :

FREQ. CONSTRAINT- 1 =	0.3274E+03
FREQ. CONSTRAINT- 2 =	0.3306E+03
FREQ. CONSTRAINT- 3 =	0.3410E+03
FREQ. CONSTRAINT- 4 =	0.3454E+03
FREQ. CONSTRAINT- 5 =	0.4208E+03
FREQ. CONSTRAINT- 6 =	0.6458E+03
FREQ. CONSTRAINT- 7 =	0.8148E+03
FREQ. CONSTRAINT- 8 =	0.8591E+03
FREQ. CONSTRAINT- 9 =	0.9196E+03
FREQ. CONSTRAINT-10 =	0.9514E+03
FREQ. CONSTRAINT-11 =	0.9538E+03

FREQ. CONSTRAINT-12 = 0.1125E+04
 FREQ. CONSTRAINT-13 = 0.1134E+04
 FREQ. CONSTRAINT-14 = 0.1142E+04
 FREQ. CONSTRAINT-15 = 0.1227E+04
 FREQ. CONSTRAINT-16 = 0.1237E+04
 FREQ. CONSTRAINT-17 = 0.1292E+04
 FREQ. CONSTRAINT-18 = 0.1298E+04
 FREQ. CONSTRAINT-19 = 0.1328E+04
 FREQ. CONSTRAINT-20 = 0.1354E+04

Do you want to respecify the initial guess ?

N

Do you have any combination constraints ?

N

RESPONSE INDICES AT THIS INITIAL CONDITION

WEIGHT INDEX @ THIS I.C = 24.6321509688000

SIZE INDEX = 0.000000000000000E+000

FREQUENCY INDEX = 3.48411480405028

DISPLACEMENT INDEX = 0.000000000000000E+000

STRESS INDEX = 0.000000000000000E+000

SMALLER THE INDEX CLOSER TO THE DESIRED RESPONSE.

Enter the weighting factors for frequency

1.

Enter the weighting factor for weight

0.

C O N M I N

FORTRAN PROGRAM FOR

CONSTRAINED FUNCTION MINIMIZATION

INITIAL FUNCTION INFORMATION

OBJ = 0.348411E+01

DECISION VARIABLES (X-VECTOR)

1) 0.10000E-03 0.10000E-03 0.10000E-03
 4) 0.10000E-03 0.10000E-03 0.10000E-03
 7) 0.10000E-03 0.10000E-03 0.10000E-03
 10) 0.10000E-03 0.10000E-03 0.10000E-03

FINAL OPTIMIZATION INFORMATION

OBJ = 0.113660E+00

DECISION VARIABLES (X-VECTOR)

1) 0.30000E+00 0.30000E+00 0.30000E+00
 4) 0.30000E+00 0.30000E+00 -0.25943E+00
 7) -0.25919E+00 0.30000E+00 0.12666E+00
 10) -0.19973E+00 -0.30000E+00 0.88716E-01

THERE ARE 7 ACTIVE SIDE CONSTRAINTS

DECISION VARIABLES AT LOWER OR UPPER BOUNDS

(MINUS INDICATES LOWER BOUND)

1 2 3 4 5 8 -11

TERMINATION CRITERION

ABS(OBJ(I)-OBJ(I-1)) LESS THAN DABFUN FOR 3 ITERATIONS

NUMBER OF ITERATIONS = 14

OBJECTIVE FUNCTION WAS EVALUATED 218 TIMES

OPTIMUM SOLUTION

FUNCTION VALUE = 0.1137E+00

NUMBER OF ITERATIONS = 14

SOLUTION :

X(1) = 0.3000E+00

X(2) = 0.3000E+00

X(3) = 0.3000E+00

X(4) = 0.3000E+00

X(5) = 0.3000E+00

X(6) = -0.2594E+00

X(7) = -0.2592E+00

X(8) = 0.3000E+00

X(9) = 0.1267E+00

X(10) = -0.1997E+00

X(11) = -0.3000E+00

X(12) = 0.8872E-01

EXPECTED RESPONSE AT THE SOLUTION:

FREQUENCIES :

FREQ. CONSTRAINT- 1	=	0.3123E+03
FREQ. CONSTRAINT- 2	=	0.3140E+03
FREQ. CONSTRAINT- 3	=	0.3307E+03
FREQ. CONSTRAINT- 4	=	0.3421E+03
FREQ. CONSTRAINT- 5	=	0.3918E+03
FREQ. CONSTRAINT- 6	=	0.6751E+03
FREQ. CONSTRAINT- 7	=	0.9263E+03
FREQ. CONSTRAINT- 8	=	0.1048E+04
FREQ. CONSTRAINT- 9	=	0.1111E+04
FREQ. CONSTRAINT-10	=	0.1174E+04
FREQ. CONSTRAINT-11	=	0.1159E+04
FREQ. CONSTRAINT-12	=	0.1395E+04
FREQ. CONSTRAINT-13	=	0.1406E+04
FREQ. CONSTRAINT-14	=	0.1250E+04
FREQ. CONSTRAINT-15	=	0.1111E+04
FREQ. CONSTRAINT-16	=	0.1142E+04
FREQ. CONSTRAINT-17	=	0.1212E+04
FREQ. CONSTRAINT-18	=	0.1204E+04
FREQ. CONSTRAINT-19	=	0.1246E+04
FREQ. CONSTRAINT-20	=	0.1466E+04
PRESENT WEIGHT	=	0.2496E+02
ORIGINAL WEIGHT	=	0.2463E+02
% CHANGE IN WEIGHT	=	0.1339E+01

8.1.2 Weight-minimization step

\$ run optimize

Enter the initial guess for these variables

0.0001

RESPONSE AT THE INITIAL GUESS:

FREQUENCIES :

FREQ. CONSTRAINT- 1 =	0.3274E+03
FREQ. CONSTRAINT- 2 =	0.3306E+03
FREQ. CONSTRAINT- 3 =	0.3410E+03
FREQ. CONSTRAINT- 4 =	0.3454E+03
FREQ. CONSTRAINT- 5 =	0.4208E+03
FREQ. CONSTRAINT- 6 =	0.6458E+03
FREQ. CONSTRAINT- 7 =	0.8148E+03
FREQ. CONSTRAINT- 8 =	0.8591E+03
FREQ. CONSTRAINT- 9 =	0.9196E+03
FREQ. CONSTRAINT-10 =	0.9514E+03
FREQ. CONSTRAINT-11 =	0.9538E+03
FREQ. CONSTRAINT-12 =	0.1125E+04
FREQ. CONSTRAINT-13 =	0.1134E+04
FREQ. CONSTRAINT-14 =	0.1142E+04
FREQ. CONSTRAINT-15 =	0.1227E+04
FREQ. CONSTRAINT-16 =	0.1237E+04
FREQ. CONSTRAINT-17 =	0.1292E+04
FREQ. CONSTRAINT-18 =	0.1298E+04
FREQ. CONSTRAINT-19 =	0.1328E+04

FREQ. CONSTRAINT-20 = 0.1354E+04

Do you want to respecify the initial guess ?

N

Do you have any combination constraints ?

N

RESPONSE INDICES AT THIS INITIAL CONDITION

WEIGHT INDEX @ THIS I.C = 24.6321509688000

SIZE INDEX = 0.000000000000000E+000

FREQUENCY INDEX = 3.48411480405028

DISPLACEMENT INDEX = 0.000000000000000E+000

STRESS INDEX = 0.000000000000000E+000

SMALLER THE INDEX CLOSER TO THE DESIRED RESPONSE.

Enter the weighting factors for frequency

1.

Enter the weighting factor for weight

.5

CONMIN

FORTRAN PROGRAM FOR

CONSTRAINED FUNCTION MINIMIZATION

INITIAL FUNCTION INFORMATION

OBJ = 0.158002E+02

DECISION VARIABLES (X-VECTOR)

1) 0.10000E-03 0.10000E-03 0.10000E-03

4) 0.10000E-03 0.10000E-03 0.10000E-03

7) 0.10000E-03 0.10000E-03 0.10000E-03

10) 0.10000E-03 0.10000E-03 0.10000E-03

FINAL OPTIMIZATION INFORMATION

OBJ = 0.113519E+02

DECISION VARIABLES (X-VECTOR)

1) 0.40873E-01 0.30000E+00 0.30000E+00

4) 0.49390E-01 0.43478E-01 -0.96453E-01

7) -0.96228E-01 0.45243E-01 -0.30000E+00

10) -0.17097E+00 -0.30000E+00 -0.30000E+00

THERE ARE 5 ACTIVE SIDE CONSTRAINTS

DECISION VARIABLES AT LOWER OR UPPER BOUNDS

(MINUS INDICATES LOWER BOUND)

2 3 -9 -11 -12

TERMINATION CRITERION

ABS(OBJ(I)-OBJ(I-1)) LESS THAN DABFUN FOR 3 ITERATIONS

NUMBER OF ITERATIONS = 11

OBJECTIVE FUNCTION WAS EVALUATED 172 TIMES

OPTIMUM SOLUTION

FUNCTION VALUE = 0.1135E+02

NUMBER OF ITERATIONS = 11

SOLUTION :

X(1) = 0.4087E-01

X(2) = 0.3000E+00

X(3) = 0.3000E+00

X(4) = 0.4939E-01

X(5) = 0.4348E-01

X(6) = -0.9645E-01

X(7) = -0.9623E-01

X(8) = 0.4524E-01

X(9) = -0.3000E+00

X(10) = -0.1710E+00

X(11) = -0.3000E+00

X(12) = -0.3000E+00

EXPECTED RESPONSE AT THE SOLUTION:

FREQUENCIES :

FREQ. CONSTRAINT- 1 = 0.3011E+03

FREQ. CONSTRAINT- 2 = 0.3081E+03

FREQ. CONSTRAINT- 3 = 0.3344E+03

FREQ. CONSTRAINT- 4 = 0.3373E+03

FREQ. CONSTRAINT- 5 = 0.3998E+03

FREQ. CONSTRAINT- 6 = 0.6822E+03

FREQ. CONSTRAINT- 7 = 0.8974E+03

FREQ. CONSTRAINT- 8 = 0.1026E+04

FREQ. CONSTRAINT- 9	=	0.1087E+04
FREQ. CONSTRAINT-10	=	0.1094E+04
FREQ. CONSTRAINT-11	=	0.1091E+04
FREQ. CONSTRAINT-12	=	0.1278E+04
FREQ. CONSTRAINT-13	=	0.1283E+04
FREQ. CONSTRAINT-14	=	0.1203E+04
FREQ. CONSTRAINT-15	=	0.1099E+04
FREQ. CONSTRAINT-16	=	0.1118E+04
FREQ. CONSTRAINT-17	=	0.1202E+04
FREQ. CONSTRAINT-18	=	0.1195E+04
FREQ. CONSTRAINT-19	=	0.1255E+04
FREQ. CONSTRAINT-20	=	0.1450E+04
PRESENT WEIGHT	=	0.2143E+02
ORIGINAL WEIGHT	=	0.2463E+02
% CHANGE IN WEIGHT	=	-0.1299E+02

§